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Original Research Article

A Mixed Integer Model for Unrelated Parallel Machine Scheduling with Job Deteriorating Effect

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Abstract

Maintenance planning has been widely applied in manufacturing systems to improve production efficiency. In some real cases, job processing times may change over time, but they are mostly assumed to be constant in the scheduling literature. Hence, in this article, a mixed integer model is developed to optimize scheduling jobs on unrelated parallel machines with reliability-based maintenance and job deteriorating effects. The proposed model considers a reliability-based maintenance system and multi-stage quality cost and starts a time-dependent deteriorating effect. Based on the assumptions, if machines work in undesirable conditions, quality reduction and quality cost increment would occur. According to the start time-dependent deteriorating effect, the processing time of each job is a function of its start time. The problem is modeled by an integer linear programming method. Computational experiments are performed on various numerical instances to show the model's effectiveness.

Keywords: Scheduling; Parallel Machine; Reliability; Maintenance; Quality.

Notations

The following notation will be used throughout this paper:

P_{im}	Processing time of job i on machine m
D_i^c	The ideal completion time (or due date) of the job i
$\propto_i$	Earliness penalty per unit time of job i
β_i	Tardiness penalty per unit time of job i
Rne_m	Necessary reliability of machine m
Rp_m	Primary reliability of machine m
Rc_m	Maintenance cost of machine m
Qc_{il}	Quality cost for each level
O_{im}	The operational cost of job i on machine m
L	A very big positive number
λ_m	The failure rate of machine m
$RL1_m$	The first reliability level of machine m
$RL2_m$	The second reliability level of machine m
a_{im}	A fixed part of the processing time for job i on machine m
b_i	The growth rate of the processing time of job i on machine m
Sets	
N	Total number of jobs
J	Total number of positions
M	Total number of available machines
Variables	
C_i	Completion time of job i
R_{mj}	Reliability of machine m in position j
$M_{mj} = \begin{cases}1\\0\end{cases}$	1 if machine m in position j is repaired; 0 otherwise
$Y_{lmj} = \begin{cases}1\\0\end{cases}$	1 if the machine m can be repaired in position j and level l; 0 otherwise
$X_{imj} = \begin{cases}1\\0\end{cases}$	1 if job i is on machine m in position j; 0 otherwise
E_i	Earliness of job i
T_i	Tardiness of job i
S_i	Start time of job i
$A_{im} = \begin{cases}1\\0\end{cases}$	1 if job i assigned to machine j;0 otherwise
D_i^s	Ideal start time of job i

1. Introduction

In an unrelated parallel machine scheduling problem, n jobs should be processed by one of the m available parallel machines. The processing time of each job depends on the machine that is assigned to process the job. Hence, each job has a different processing time on each machine [1]. While parallel machine scheduling has been broadly discussed in the literature, unrelated parallel machine scheduling has rarely been discussed. To solve unrelated parallel machine scheduling, heuristic and Tabu search algorithms with the objective of minimizing the total weighted tardiness and the maximum lateness are proposed by Kim et al. [2] and Kim and Shin [3], respectively. Unrelated parallel machine scheduling with resource constraints is studied by Chen [4] and Chen and Wu [5]. Kayvanfar et al. [6] studied unrelated parallel machines with controllable processing times to minimize total tardiness and earliness. It can be seen that in previous studies, tardiness and earliness are considered objective functions, and heuristic and metaheuristic algorithms are applied to solve the model. Berthier et al. [7] considered an unrelated parallel machines scheduling problem with the machine and sequence-dependent setup times, machine eligibility, and different resource types constraints. Al-qaness et al. [8] introduced a new method to address the Unrelated parallel machine scheduling problem with sequence-dependent and machine-dependent setup time

A common assumption in most machine scheduling problems is that the machines are always available to process the jobs. This assumption is unreasonable in several situations where machines must be serviced during the planning horizon. Hence, it is crucial to consider job scheduling and maintenance planning simultaneously to reduce unplanned maintenance actions, maintenance time, failure rate, tardiness and related costs. In the literature on machine scheduling, maintenance is frequently treated as a machine scheduling and availability constraint [9]. Schmidt [10] studied parallel machine scheduling where each machine has the same speed, but availability intervals are different. Adiri et al. [11] and Lee and Liman [12] studied the problem of single-machine scheduling to minimize the flow time by considering breakdowns during the process. Lee and Chen [13] proposed a parallel machine model where only one maintenance activity is allowed during the planning horizon.

Recently, some researchers have considered a combination of scheduling with maintenance or quality, but a few of them have considered all of them simultaneously. Linderman et al. [14] coordinated the maintenance and process control decisions by proposing a model to determine the optimal policy. Panagiotidou and Tagaras [15] presented an economic model to optimize preventive maintenance with consideration of two quality states, the in-control state and the out-of-control state. Pandey et al. [16] developed a model to optimize maintenance planning, process quality, and production scheduling with consideration of quality cost. Jamshidi and Seyyed Esfahani [17] proposed a parallel machine scheduling model to optimize the quality cost, maintenance cost, earliness-tardiness cost, and interruption cost simultaneously.

In real-world applications, jobs may deteriorate while waiting to be processed. Job deterioration implies that the processing time of each job is a function of its position or/and start time. For instance, when an ingot is waiting to be processed by a rolling machine, its temperature will be decreased; thus, the ingot should be reheated before entering the rolling machine. These sorts of problems are known as deteriorating job scheduling [18], which was introduced by Browne and Yechiali [19], and several recent studies have incorporated the deteriorating effect [20]. Raut et al. [21] studied a single-machine scheduling problem with consideration of deteriorating jobs and limited capacity for machines to maximize total revenue. Huang and Wang [22] studied parallel machine scheduling with deteriorating jobs. In their article, they concentrated on two goals, minimizing the total absolute difference between waiting for time (TADW) and completion time (TADC). Mazdeh et al. [18] proposed a parallel machine scheduling model to minimize job tardiness and deteriorating machine cost with the deteriorating job.

According to the literature, there are several studies that have considered maintenance planning, quality considerations, and job deteriorating effect in the unrelated parallel machine environment. However, this is the first study that takes all the above-mentioned aspects into account simultaneously. The rest of the paper is organized as follows: In Section 2, the assumptions and the mathematical model are presented. In section 3, a sensitivity analysis has been fulfilled in order to validate the proposed model. In Section 4, numerical examples are presented to show the applicability and effectiveness of the model. Finally, Section 5 is dedicated to the conclusion and future research directions.

2. Problem formulation

The problem of this study can be described as follows**:** There is a set $J = \{j_1, j_2, \ldots, j_n\}$ of n jobs to be scheduled on m parallel machines with a reliability based-maintenance system. All jobs are available at time zero, and preemption is allowed. Job deterioration is assumed in which job processing time is a function of the starting time and fixed part of the processing time ($P_{im} \geq a_{im} + [b_i \times S_i]$). In addition, a bi-stage quality cost is considered. In the reliability-based maintenance system, the exponential distribution is considered for all machines' failures. Figure 1 shows the reliability levels. It is assumed that each machine has two reliability levels and if the reliability of a machine falls below the first level (RL_1) due to the reliability value and two-stage quality cost, a certain amount of cost would be incurred. On the other hand, when machine reliability falls below the second level (RL_2), it has to be repaired. Noteworthy to mention, the proposed model is inspired by Jamshidi and Seyyed Esfahani [17], Chen and Wu [5], and Mazdeh et al. [18].

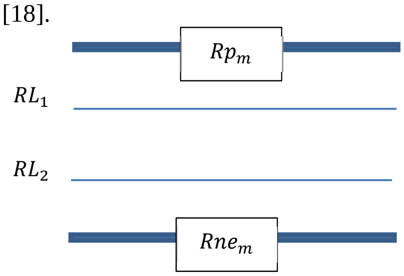

Figure 1. The reliability levels

3. Mathematical model

The unrelated parallel machine-scheduling model, by considering reliability-based maintenance and job deterioration can be formulated as a mixed nonlinear integer programming model as follows:

$$Min\ Z = \sum_{i=1}^{N}(\propto_i E_i + \beta_i T_i) + \sum_{i=1}^{N}\sum_{l=1}^{l}[(\sum_{m=1}^{M}\sum_{j=1}^{J}\frac{X_{imj}.Y_{lmj}}{P_{im}}).Qc_{il}] + \sum_{m=1}^{M}\sum_{j=1}^{J}Rc_m.M_{mj} + \sum_{i=1}^{I}\sum_{m=1}^{M}A_{im}O_{im} \quad (1)$$

$S.t.$

$$\sum_{j=1}^{J}X_{imj} = P_{im}.A_{im} \qquad \forall\ i\epsilon I\ m\epsilon M \quad (2)$$

$$\sum_{m=1}^{M}X_{imj} + M_{mj} \le 1 \qquad \forall\ m\epsilon M, j\epsilon J \quad (3)$$

$$\frac{\sum_{j=1}^{J}X_{imj}}{P_{im}} \le A_{im} \qquad \forall\ i\epsilon I\ m\epsilon M \quad (4)$$

$$\sum_{m=1}^{M}A_{im} = 1 \qquad \forall\ i\epsilon I \quad (5)$$

$$\frac{1}{2}\sum_{m=1}^{M}(\sum_{j=1}^{j-1}|X_{imj} - X_{im(j+1)}|) + X_{im1} + X_{imj}) - 2) = 0 \qquad \forall\ i\epsilon I \quad (6)$$

$$RL_2 - R_{mj} \le Y_{2mj} \qquad \forall\ m\epsilon M, j\epsilon J \quad (7)$$

$$Y_{2mj} \le \max\{0, L.(RL2_m - R_{mj})\} \qquad \forall\ m\epsilon M, j\epsilon J \quad (8)$$

$$RL_1 - R_{mj} \le Y_{1mj} + L.Y_{2mj} \qquad \forall\ m\epsilon M, j\epsilon J \quad (9)$$

$$Y_{1mj} \le \max\{0, L.(RL1_m - R_{mj})\} \qquad \forall\ m\epsilon M, j\epsilon J \quad (10)$$

$$Y_{2mj} \le 1 - Y_{1mj} \qquad \forall\ m\epsilon M, j\epsilon J \quad (11)$$

$$Rne_m - R_{mj} \le M_{mj} \qquad \forall\ m\epsilon M, j\epsilon J \quad (12)$$

$$M_{mj} \le \sum_{l=1}^{l}Y_{lmj} \qquad \forall\ m\epsilon M, j\epsilon J \quad (13)$$

$$R_{mj} = (R_{m,j-1}).e^{-\lambda}.(1 - M_{m.j-1}).(\sum_{i=1}^{N}X_{im,j-1}) + Rp_m.M_{m.j-1} + (R_{m,j-1}).(1 - M_{m.j-1}).(1 - \sum_{i=1}^{N}X_{im,j-1}) \qquad \forall\ m\epsilon M, j\epsilon J \quad (14)$$

$$P_{im} \ge a_{im} + \lceil b_i \times S_i \rceil \qquad \forall\ i\epsilon I\ m\epsilon M \quad (15)$$

$$T_i \ge C_i - D_i^c \qquad \forall\ i\epsilon I \quad (16)$$

$$E_i \ge D_i^s - S_i \qquad \forall\ i\epsilon I \quad (17)$$

$$C_i = \max_m[\max_j(j \times X_{imj})] \qquad \forall\ i\epsilon I \quad (18)$$

$$S_i = \min_m[\min_j[j + A(1 - X_{imj})]] \qquad \forall\ i\epsilon I \quad (19)$$

$$D_i^s = D_i^c - \sum_{m=1}^{M}P_{im}.A_{im} + 1 \qquad \forall\ i\epsilon I \quad (20)$$

$$X_{imj}, M_{mj}, Y_{lmj}, A_{im}\ \epsilon\ \{0,1\} \qquad \forall\ i\epsilon I\ m\epsilon M \quad (21)$$

$$C_i, S_i, T_i, E_i \ge 0 \qquad \forall\ i\epsilon I \quad (22)$$

The objective function (1) minimize total costs, including (i) total weighted earliness and tardiness cost, (ii) total repair cost, (iii) total multi-stage quality cost, and (iv) total cost of job processing. Equality (2) assures that the total processing time of job i is equal to P_{im}. Constraint (3) indicates that when a machine is working in position j, a repair cannot be executed on the machine. Equations (4) and (5) together ensure that each job must be assigned to one machine. Constraint (6) indicates that preemption is not allowed. Constraints (7) to (11) state that if $RL_1 < R_{mj} < RL_2$, then Y_{1mj} must be equal to 1, and if $RL_2 < R_{mj} < Rne_m$, then Y_{2mj} must be equal to 1. Constraint (12) guarantees that a machine must be repaired when its reliability falls below the necessary reliability. Constraint (13) ensures that a machine is repaired when it is allowed to be repaired. Equality (14) calculates the reliability of each machine in each position due to its previous status. If a machine is worked in position $j-1$, the reliability of the machine would be decreased based on its failure rate. If a machine is repaired in position $j-1$, the reliability of the machine would be increased to the primary level; and in case of being idle in position $j-1$, it is obvious that the reliability would be the same as before. Constraint (15) dedicates to the job deterioration effect in which the processing time of each machine depends on its start time. Constraints (16) and (17) calculate tardiness and earliness values for each job. Equations (18) and (19) calculate the completion time and start time of each job, respectively. Equality (20) calculates the ideal start time of each job. Sets (21) and (22) define the variables.

3.1 Linearization of the model

Since the proposed model has a non-linear component, most of the non-linear terms are converted to linear ones using the linearization method proposed by Jamshidi and Seyyed Esfahani [17]. However, some minor terms remain non-linear. Firstly, the quality cost in the objective function is linearized by using the variable $XY_{ilmj} = X_{imj}.Y_{lmj}$ under the following constraints:

$$XY_{ilmj} \geq Y_{lmj} - L(1 - X_{imj}) \quad \forall\ i,l,m,j \quad (23)$$
$$XY_{ilmj} \leq Y_{lmj} + L(1 - X_{imj}) \quad \forall\ i,l,m,j \quad (24)$$
$$XY_{ilmj} \leq X_{imj} \quad \forall\ i\epsilon I \quad (25)$$
$$XY_{ilmj} \geq 0 \quad \forall\ i,l,m,j \quad (26)$$

Equation (2) could be linearized by using the variable $PA_{im} = P_{im}.A_{im}$ and add the following constraints:

$$PA_{im} \geq P_{im} - L(1 - A_{im}) \quad \forall\ i,m \quad (27)$$
$$PA_{im} \leq P_{im} - L(1 - A_{im}) \quad \forall\ i,m \quad (28)$$
$$PA_{im} \leq L.A_{im} \quad \forall\ i,m \quad (29)$$
$$PA_{im} \geq 0 \quad \forall\ i,m \quad (30)$$

Equation (6), which indicates the number of interruptions must be 0 for each job, is transformed to linear form in Equation (31) by replacing the term $|X_{imj} - X_{im(j+1)}|$ to $XP_{imj} + XM_{imj}$ and under constraints (32) and (33):

$$\frac{1}{2}\sum_{m=1}^{M}((\sum_{j=1}^{j-1}(XP_{imj} + XM_{imj})) + X_{im1} + X_{imJ}) - 2 = 0 \quad \forall\ i,m,j \quad (31)$$
$$X_{imj} - X_{im(j+1)} = XP_{imj} - XM_{imj} \quad \forall\ i,m,j \quad (32)$$
$$XP_{imj}, XM_{imj} \geq 0 \quad \forall\ i,m,j \quad (33)$$

Constraints (8) and (10) could be linearized by the following constraints:

$$Y_{2mj} \leq 1 - (R_{mj} - RL2_m) \quad \forall\ m,j \quad (34)$$
$$Y_{1mj} \leq 1 - (R_{mj} - RL1_m) \quad \forall\ m,j \quad (35)$$

Equation (14) is transformed to equation (36) by replacing variable $RM_{mj} = R_{mj}.M_{mj}$ and adding constraints $(37)-(40)$:

$$R_{mj} = \left(R_{m,j-1} - RM_{m,j-1}\right).(\sum_{i=1}^{N} X_{im,j-1}).e^{-\lambda} + Rp_m.M_{m.j-1}\left(R_{m,j-1} - RM_{m,j-1}\right).(1 - \sum_{i=1}^{N} X_{im,j-1}) \quad \forall\ m,j \quad (36)$$
$$RM_{mj} \geq R_{mj} - L(1 - M_{mj}) \quad \forall\ i,m \quad (37)$$
$$RM_{mj} \leq R_{mj} + L(1 - M_{mj}) \quad \forall\ i,m \quad (38)$$
$$RM_{mj} \leq M_{im} \quad \forall\ i,m \quad (39)$$
$$RM_{mj} \geq 0 \quad \forall\ i,m \quad (40)$$

Since Equation (36) is still non-linear, variables $RX_{mj} = R_{mj}.\sum_i X_{imj}$ and $RMX_{mj} = RM_{mj}.\sum_i X_{imj}$ are replaced to transform it to linear form by the following set of constraints:

$$R_{mj} = \left(RX_{m,j-1} - RMX_{m,j-1}\right).e^{-\lambda} + Rp_m.M_{m.j-1} + \left(R_{m,j-1} - RM_{m,j-1}\right) - \left(RX_{m,j-1} - RMX_{m,j-1}\right) \quad \forall\ m,j \quad (41)$$
$$RMX_{mj} \geq RM_{mj} - L(1 - \sum_i X_{imj}) \quad \forall\ m,j \quad (42)$$
$$RMX_{mj} \leq RM_{mj} + L(1 - \sum_i X_{imj}) \quad \forall\ m,j \quad (43)$$
$$RMX_{mj} \leq L.\sum_i X_{imj} \quad \forall\ m,j \quad (44)$$
$$RX_{mj} \geq R_{mj} - L(1 - \sum_i X_{imj}) \quad \forall\ m,j \quad (45)$$
$$RX_{mj} \leq RM_{mj} + L(1 - \sum_i X_{imj}) \quad \forall\ m,j \quad (46)$$
$$RMX_{mj} \leq L.\sum_i X_{imj} \quad \forall\ m,j \quad (47)$$
$$RX_{mj}, RMX_{mj} \geq 0 \quad \forall\ m,j \quad (48)$$

For Constraints (15), variable (BS_i) is defined to remove the term $(\lceil b_i \times S_i \rceil)$. Hence, constraints (15) must be replaced by constraints (49). Constraint (50) calculates the variable (BS_i) :

$$P_{im} \geq a_{im} + BS_i \quad \forall\ i,m \quad (49)$$
$$BS_i \geq b_i \times S_i \quad \forall\ i \quad (50)$$
$$BS_i \in integer \quad \forall\ i \quad (51)$$

Since the objective function of the proposed model is minimization, obtained value for (BS_i) in constraint (50) is exactly equal to the term $(\lceil b_i \times S_i \rceil)$. To remove, the non-linear term in Equations (18) and (19), at first, constraints (16) and (17) are replaced by the following constraints:

$$T_i \geq Q_{iJ} - D_i^c \quad \forall\ i \quad (49)$$
$$E_i \geq (D_i^c - P_i) - (J - \sum_{j=1}^{J} B_{ij}) \quad \forall\ i \quad (50)$$

Then, Q_{ij} and B_{ij} are formulated as follows:

$$Q_{iJ} = Q_{iJ-1}(1 - \sum_{m=1}^{M} X_{imj}) + (j\sum_{m=1}^{M} X_{imj}) \quad \forall\ i,j \quad (51)$$
$$B_{iJ} = B_{iJ-1} + (1 - B_{iJ-1})\sum_{m=1}^{M} X_{imj} \quad \forall\ i,j \quad (52)$$

Since Equations (51) and (52) are still non-linear, auxiliary variables are defined to transform the aforementioned equations to linear form as follows:

$$F_{ij} = j\sum_{m=1}^{M} X_{imj}$$
$$V_{ij} = Q_{ij-1}\sum_{m=1}^{M} X_{imj}$$
$$G_{ij} = B_{ij-1}\sum_{m=1}^{M} X_{imj}$$

These auxiliary variables are defined under constraints $(53)-(62)$.

$$F_{ij} + L(\sum_{m=1}^{M} X_{imj} - 1) \leq j \quad \forall\ i,j \quad (53)$$
$$F_{ij} - L(\sum_{m=1}^{M} X_{imj} - 1) \geq j \quad \forall\ i,j \quad (54)$$
$$F_{ij} \leq L\sum_{m=1}^{M} X_{imj} \quad \forall\ i,j \quad (55)$$
$$V_{IJ} + L(\sum_{m=1}^{M} X_{imj} - 1) \leq Q_{IJ-1} \quad \forall\ i,j \quad (56)$$
$$V_{IJ} - L(\sum_{m=1}^{M} X_{imj} - 1) \geq Q_{IJ-1} \quad \forall\ i,j \quad (57)$$
$$V_{IJ} \leq L\sum_{m=1}^{M} X_{imj} \quad \forall\ i,j \quad (58)$$
$$G_{IJ} + L(\sum_{m=1}^{M} X_{imj} - 1) \leq B_{IJ-1} \quad \forall\ i,j \quad (59)$$
$$G_{IJ} - L(\sum_{m=1}^{M} X_{imj} - 1) \geq B_{IJ-1} \quad \forall\ i,j \quad (60)$$
$$G_{IJ} \leq L\sum_{m=1}^{M} X_{imj} \quad \forall\ i,j \quad (61)$$

$$Q_{IJ}, B_{IJ}, F_{ij}, V_{IJ}, G_{IJ} \geq 0 \qquad \forall\ i,j \qquad (62)$$

Therefore, constraints (51) and (52) are transformed to linear form as shown by equations (63) and (64).

$$Q_{iJ} = Q_{iJ-1} - V_{ij} + F_{ij} \qquad \forall\ i,j \qquad (63)$$

$$B_{iJ} = B_{iJ-1} + \sum_{m=1}^{M} X_{imj} - G_{ij} \qquad \forall\ i,j \qquad (64)$$

Finally, the mathematical model is introduced as follow:

$$Min\ Z = \sum_{i=1}^{N} (\propto_i E_i + \beta_i T_i) + \sum_{m=1}^{M} \sum_{j=1}^{J} Rc_m . M_{mj} + \sum_{i=1}^{N} \sum_{l=1}^{3} [(\sum_{m=1}^{M} \sum_{j=1}^{J} \frac{XY_{ilmj}}{P_{im}}) . Qc_{il}] + \sum_{i=1}^{I} \sum_{m=1}^{M} A_{im} C_{im}$$

S.t

Constraints $(23) - (26), (27) - (30), (3) - (5), (31) - (33), (7), (9), (34), (35), (11) - (13), (37) - (48), (15), (49), (50), (53) - (64)$

4. Results

The model presented in Section (2) is a mixed-integer non-linear programming optimization problem. according to non-linear constraint (4) and term $(XY_{ilmj}/P(i,m))$ in the objective function. The proposed model has been coded in GAMS 24.1 software by using the BONMIN solver. The reliability and validity of the proposed model are proved by numerical instances. A Pentium 4 computer with a 2.5 GHz CPU and 6 GB RAM is used to run the model.

Since test problems on parallel machine scheduling by considering reliability-based maintenance and job deterioration have not been presented in the literature, test problems are generated with various numbers of machines (2 and 3) and jobs (5, 7, 9, and 11). Other parameters, including $(\propto_i)$, (β_i), (Rp_m), (Rc_m), (λ_m) are generated based on Jamshidi and Seyyed Esfahani [17]. Tables 1 and Table 2 show related data. In Table 1, a fixed processing time matrix is created using discrete uniform distribution [3,8], and jobs processing costs are also produced by uniform distribution [1000,1200]. Earliness and tardiness penalties are generated by uniform distributions on the intervals [17,23] and [15,25], respectively. The growth rate in this table is obtained by discrete uniform distribution on the interval [0,2]. Also, the quality cost matrix for levels one and two are generated by uniform distribution on the intervals [420,360] and [240,280], respectively. Table 2 shows machine parameters.

Table 3 provides the due dates of jobs for two and three machines.

Table 1. Randomly generated parameters for eleven jobs

Job parameters												
Jobs		i=1	i=2	i=3	i=4	i=5	i=6	i=7	i=8	i=9	i=10	i=11
a_{im}	m=1	4	5	3	5	4	7	5	5	6	4	4
	m=2	7	6	8	5	6	7	7	8	6	4	8
	m=3	4	7	7	5	6	6	6	8	5	5	4
C_{im}	m=1	1162.9	1182.7	1055.7	1193	1191.4	1028.4	1158.4	1007.1	1135.7	1078.4	1141.2
	m=2	1181.2	1126.5	1109.4	1031.5	1097.1	1084.4	1191.9	1169.8	1151.5	1131.1	1006.4
	m=3	1025.4	1019.5	1191.5	1194.1	1160.1	1183.1	1131.2	1186.8	1148.6	1034.2	1055.4
$\propto_i$		21.07	20.93	17.97	17.71	19.98	22.75	19.04	20.51	18.34	21.50	18.53
β_i		17.43	24.29	18.49	16.96	17.51	21.16	19.73	18.51	23.30	20.85	20.49
b_i		0.109	0.127	0.154	0.195	0.196	0.109	0.127	0.154	0.195	0.142	0.115

Table 2. Parameters for three machines

Machine parameters			
m	1	2	3
Rc_m	547.97	493.88	461.11
Rp_m	0.916	0.901	0.944
$RL1$	0.831	0.804	0.82
$RL2$	0.746	0.707	0.696
Rne_m	0.66	0.61	0.57
λ_m	0.023	0.019	0.038

Table 3. Due dates for eleven jobs

Due dates											
Jobs	i=1	i=2	i=3	i=4	i=5	i=6	i=7	i=8	i=9	i=10	i=11
m=2	30	35	50	65	75	80	85	100	105	115	120
m=3	20	25	35	45	50	55	60	70	75	85	90

Computational results for each instance, which consist of tardiness, earliness and total cost, are presented in Table 4 to Table 11. Also, related job assignment for each test problem is presented.

Table 4. Result of the test problem with five jobs and two machines

Instance (1)	r	1	2	3
$M1$	i	1	4	-
$M2$	i	2	3	5
Total Cost		7454.55		

Table 5. Result of the test problem with five jobs and three machines

Instance (2)	r	1	2	3
$M1$	i	1	4	-
$M2$	i	2	3	-
$M3$	i	5	-	-
Total Cost		6236.78		

Table 4 and 5, shows the results of the test problem with 5 jobs. In Table 4, jobs are scheduled on two machines, whereas in Table 5, three machines are used to schedule the jobs. In the case of two machines (Table 4), jobs 1 and 2 are assigned to machine 1, and jobs 2, 3, and 5 are assigned to machine 2. When there are three machines, the sequence is the same, except that job 5 is assigned to machine 3 (Table 5). Adding a new machine resulted in a total cost reduction of 16%.

Table 6 shows the solution for the problem with seven jobs and two machines, and Table 6 shows the solution for the problem with seven jobs and three machines. The job sequence between the two test problems (Table 6 and Table 7) is more diverse in comparison to Tables 4 and 5, where there was only a slight difference between the two test problems. Adding a new machine in Table 7 resulted in a total cost reduction of 5%.

Table 6. Result of the test problem with seven jobs and two machines

Instance (3)	r	1	2	3	4	5
$M1$	i	5	7	-	-	-
$M2$	i	3	1	2	4	-
Total Cost			11158.55			

Table 7. Result of the test problem with seven jobs and three machines

Instance (4)	r	1	2	3	4	5
$M1$	i	2	3	-	-	-
$M2$	i	4	1	5	-	-
$M3$	i	6	7	-	-	-
Total Cost			10624.13			

Table 8. Result of the test problem with nine jobs and two machines

Instance (5)	r	1	2	3	4	5
$M1$	i	8	3	9	7	2
$M2$	i	6	5	1	4	-
Total Cost			16458.55			

Table 9. Result of the test problem with nine jobs and three machines

Instance (6)	r	1	2	3	4	5
$M1$	i	3	7	9	-	
$M2$	i	6	1	-	-	
$M3$	i	5	4	2	8	
Total Cost			14231.78			

In Table 8, which shows the results of the test problem with nine jobs on two machines total cost is 16458.55, and in Table 9, which shows the results of the test problem with nine jobs but on three machines total cost is 14231.78. In these two test problems, due to the increase in the number of jobs that must be scheduled, earliness and tardiness costs are more significant in comparison to the previous test problems. It is obvious that the costs for the test problem with two machines are more than the case with three machines. As can be seen, the objective function would increase when the number of jobs and the required level of reliability increase. Figure 2 depicts the relation between computation time and the number of jobs and machines.

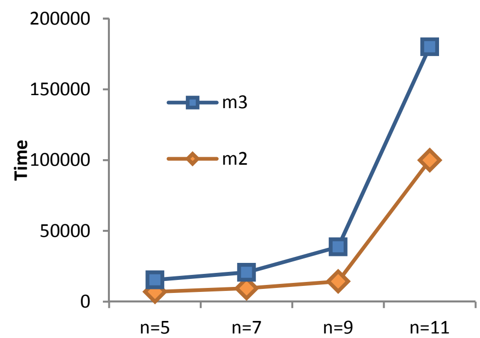

Figure 2. Computation time (second) for different test problems

It can be seen that there is a direct relation between the number of jobs and machines and computation time; especially when the number of jobs becomes more than 9, a sharp rise in computation time occurs.

4.1 Sensitivity analysis

In this section, several sensitivity analyses are carried out on two significant parameters of the proposed model: the growth rate of the processing time (b_i) and quality cost (Q_{il}). Figures 3 and 4 depict the impact of the parameter alteration on the objective function. It should be noted that sensitivity analysis is done for test problem 2.

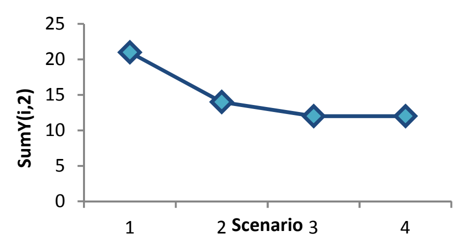

Figure 3. Total numbers of Y (2, m, j) in different scenarios

Figure 3, depicts the sensitivity analysis on a different value of the quality cost of level 2 during four scenarios. In the second scenario, the quality costs of level 2 for different machines are according to table 1. In the first scenario, the quality costs of level 2 are 150 units less than the first scenario and in the third and fourth scenarios quality costs of level 2 are 200 and 250 units more than the first scenario, respectively. It can be seen in fig 3 that as the cost becomes lower, the total number of the positions in which the system incurs the quality cost of level 2 decreases. On the other hand, increasing the quality cost at this level lets the system incur more positions in which quality costs are active in them.

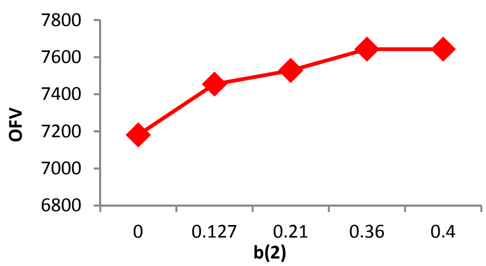

Figure 4. Objective function vs. the growth rate of job 2

Figure 4 shows the sensitivity analysis on a different value of the growth rate parameter of job 2. The processing time of each job is a function of a fixed time, start time, and growth rate. Increasing the growth rate of job results in more processing time for the job under two conditions; If the job is not sequenced at the first time interval and at the same time, the value of the growth rate increases sufficiently due to the change in the value of term $\lceil S_i \times b_i \rceil$. In this case, the objective function increases in each scenario due to the increasing growth rate value for job 2, from 0 to 0.127, 0.127 to 0.21 and 0.21 to 0.36. But it remains the same by increasing the value of the growth rate from 0.36 to 0.4 because this amount of change did not change the value of the term $(\lceil S_i \times b_i \rceil)$.

As can be seen in Table 10, in this study reliability-based maintenance model is proposed for unrelated parallel machines by considering the quality and operating costs, tardiness and earliness, job deterioration, and preemption.

Table 10. The features of this paper versus some recent works.

Paper	Environment		Maintenance			Quality Cost	Tardiness and earliness	Job deterioration	Preemption	Different Operating Cost
	Parallel machine	Other	Reliability-base maintenance	Other	None					
Cheng et al. [23]							✓	✓		
Panagiotidou and Tagaras [15]						✓				
Berrichi et al. [24]	•		•							
Mazdeh et al. [18]	•						✓	✓		✓
(Jamshidi and Seyyed Esfahani [17])	•		•			✓	✓		✓	
Current study	•		•			✓	✓	✓	✓	✓

5. Conclusions

In this paper, a mixed non-linear integer-programming model is proposed to schedule jobs by considering reliability-based maintenance. Considering the effects of job deterioration and multi-stage quality cost make the scheduling jobs model more realistic, and therefore, the deterioration effect on parallel machines is incorporated. The proposed model assigns jobs to the machines in the best position to minimize earliness and tardiness penalty costs, maintenance costs, operational costs, and quality costs that are incurred because of poor quality of products. The performance of the proposed model has been evaluated, and the applicability of the model is shown on some randomly generated data sets. Due to the difficulty in obtaining an optimal solution for large-sized problems in reasonable computational time, our future work will implement an efficient meta-heuristic algorithm to solve this type of parallel machine scheduling problem. Furthermore, since reliability-based maintenance and other types of machine scheduling such as flow shop, job shop, and open shop have not been considered, considering reliability-based maintenance in the aforementioned environments can be done in future studies.

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