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Original Research Article

Fault Detection of a Quadrotor Actuator Based on the Luenberger Estimation Method

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Abstract

Quadrotors, as nonlinear and multivariable flight systems, are inherently susceptible to various actuator faults that may compromise their performance and stability. In this study, a method based on the Luenberger observer algorithm is proposed for the detection and identification of actuator faults in quadrotors. First, the system's dynamic model is derived using standard approaches, and then linearized to facilitate the observer design. The Luenberger algorithm is developed based on the linearized model, and its accuracy is assessed through comparison with the nonlinear model. To evaluate the performance of the proposed algorithm, specific actuator faults are intentionally introduced into one of the system's actuators. Simulation results show that the proposed method can effectively identify the induced faults with high accuracy and a rapid response. This capability can significantly contribute to the development of robust control systems for critical quadrotor applications.

Keywords: Fault Detection; Luenberger; Quadrotor; Quadrotor Fault Diagnosis; State Estimation.

Nomenclature

A	State matrix of the linearized system
B	Input matrix of the linearized system
C	Output matrix of the linearized system
b	Thrust coefficient $(N \cdot s^2)$
d	Drag coefficient $(N \cdot m \cdot s^2)$
e	Estimation error vector, $e = x - \hat{x}$
$f(\cdot,\cdot)$	Nonlinear state function, $\dot{x} = f(x,u)$
Ixx	Moment of inertia about $x-$axis $(kg \cdot m^2)$
Iyy	Moment of inertia about $y-$axis $(kg \cdot m^2)$
Izz	Moment of inertia about $z-$axis $(kg \cdot m^2)$
L	Observer gain matrix (Luenberger gain)
PDF	Probability Density Function
t	Time (s)
UAV	Unmanned Aerial Vehicle
u	Control input vector
$U1$	Total thrust (N)
$U2$	Control torque about roll axis $(N \cdot m)$
$U3$	Control torque about pitch axis $(N \cdot m)$
$U4$	Control torque about yaw axis $(N \cdot m)$
X	State vector, $X = [p, v, \phi, \theta, \psi, \dot{p}, \dot{\theta}, \dot{\psi}]^T$
y	Measured output vector
ϕ	Roll angle (deg)
θ	Pitch angle (deg)
ψ	Yaw angle (deg)
ω	Angular velocity vector (rad/s)

1. Introduction

Quadrotors, as one of the most widely utilized categories of aerial robots, have attracted substantial attention from researchers due to their high maneuverability, simple mechanical structure, and broad operational versatility across various aerial missions [1]. They are commonly employed in applications such as surveillance, service delivery, emergency response, and the inspection of pipelines and oil facilities [2].

However, the inherent dynamic complexities and pronounced nonlinear behavior of quadrotor systems present substantial challenges for both control system design and fault detection [3, 4]. Among the various degradation mechanisms, actuator faults constitute a primary source of instability and performance deterioration [5]. Therefore, accurate fault detection, coupled with the development of robust mitigation strategies, is essential to ensure reliable and stable operation [6].

Faults in quadrotor systems generally occur in actuators or sensors [7]. Actuator malfunctions can impair overall system performance and, in severe cases, lead to complete operational failure. As a result, numerous fault detection methodologies have been

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developed for such platforms [8]. For example, in [9], a Luenberger observer was employed to construct a model for identifying sensor-induced faults, followed by the synthesis of a compensatory controller. It is worth noting that this controller was designed based on a linearized representation of the system dynamics.

In [10], the system's dynamic model was thoroughly examined, and the proposed algorithm was implemented to detect and evaluate actuator faults. The results indicated that the method achieved highly accurate fault estimation and contributed substantially to improving system performance and stability. Similarly, in [11], a robust control strategy was proposed for fault detection and system reconfiguration in quadrotors, with the aim of enhancing both operational efficiency and reliability. In addition, intelligent and innovative approaches have been explored as tools for fault detection across various systems. For instance, in [12], a neural network–based scheme was applied for actuator fault detection in quadrotors; however, a major drawback of such techniques is their predominantly offline implementation.

In this study, a Luenberger observer–based framework is employed to estimate the system states and detect actuator faults in a quadrotor platform. The approach begins with a linear actuator model, followed by the estimation of the nonlinear system dynamics under actuator fault conditions, thereby enabling fault identification. Simulation results confirm the effectiveness of the proposed methodology.

The structure of the paper is organized as follows: Section 2 presents the nonlinear dynamics of the quadrotor and its linearized model used for observer design. Section 3 describes the design of the Luenberger observer and the fault detection methodology. Section 4 discusses the simulations and results obtained under actuator fault scenarios. Finally, Section 5 provides conclusions along with recommendations for future work.

2. Dynamic Model of Quadrotor

In this section, the nonlinear dynamic model of the quadrotor system, illustrated in Figure 1, is presented. The governing equations are formulated in state–space representation, following the methodology outlined in [13]. The state vector X is expressed by partitioning it into translational and rotational components as follows:

$$\begin{aligned}
\ddot{\phi} &= \dot{\theta}\dot{\psi}\left(\frac{(I_{yy}-I_{zz})}{I_{xx}}\right) - \dot{\theta}\,\frac{I_{rotor}}{I_{xx}}\Omega_r + \frac{u_2}{I_{xx}} \\
\ddot{\theta} &= \dot{\phi}\dot{\psi}\left(\frac{(I_{zz}-I_{xx})}{I_{yy}}\right) - \dot{\phi}\,\frac{I_{rotor}}{I_{yy}}\Omega_r + \frac{u_3}{I_{yy}} \\
\ddot{\psi} &= \dot{\phi}\dot{\theta}\left(\frac{(I_{xx}-I_{yy})}{I_{zz}}\right) + \frac{u_4}{I_{zz}} \\
\ddot{x} &= (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)\frac{1}{m}u_1 \\
\ddot{y} &= (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)\frac{1}{m}u_1 \\
\ddot{z} &= -g + (\cos\phi\cos\theta)\frac{1}{m}u_1
\end{aligned} \tag{1}$$

Here, by separating the transition part from the state, the vector X is:

$X = [\,\phi\;\dot{\phi}\;\;\theta\;\dot{\theta}\;\;\psi\;\dot{\psi}\,]^T$

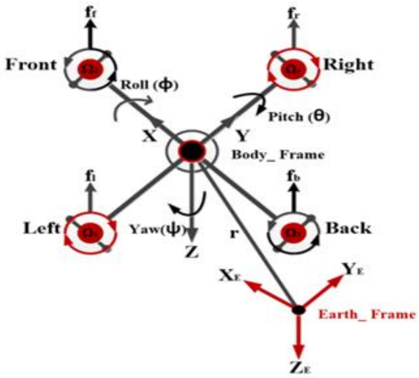

Figure 1. Quadrotor coordinate system

Therefore, these Equations can be rewritten in closed form as written in [14]

$$\dot{X} = F(X,U) = \begin{bmatrix} \dot{\emptyset} \\ \dot{\theta}\dot{\psi}a_1 + b_1U_2 \\ \dot{\theta} \\ \dot{\phi}\dot{\psi}a_3 + b_2U_3 \\ \dot{\psi} \\ \dot{\theta}\dot{\phi}a_5 + b_3U_4 \end{bmatrix} \tag{2}$$

In this formulation, the state variables are represented by the vector X, while the input variables are represented by the vector U. The state vector encompasses the translational positions, linear velocities, Euler angles $(\emptyset, \theta, \psi)$, and their corresponding angular rates $(\dot{\emptyset}, \dot{\theta}, \dot{\psi})$. The remaining variables of the system are defined as follows:

$$\begin{aligned}
&a_1 = \frac{(I_{yy}-I_{zz})}{I_{xx}} \quad a_2 = \frac{Jr}{I_{xx}} \quad a_3 = \frac{(I_{zz}-I_{xx})}{I_{yy}} \quad a_4 = \frac{Jr}{I_{yy}} \\
&a_5 = \frac{(I_{xx}-I_{yy})}{I_{zz}} \\
&U_2 = b\,(-\Omega_2^{\,2} + \Omega_4^{\,2}) \\
&U_3 = b\,(\Omega_1^{\,2} - \Omega_3^{\,2}) \\
&U_4 = d\,(-\Omega_1^{\,2} + \Omega_2^{\,2} - \Omega_3^{\,2} + \Omega_4^{\,2}) \\
&b_1 = \frac{l}{I_{xx}} \quad b_2 = \frac{l}{I_{yy}} \quad b_3 = \frac{1}{I_{zz}}
\end{aligned} \tag{3}$$

Where b is the thrust coefficient and d is the drag coefficient.

Subsequently, in preparation for applying the Luenberger estimation methodology, the quadrotor dynamic equations are linearized via a first-order Taylor series expansion. In this context, the nonlinear representation $F(X,U)$ is transformed into its linear form, thereby facilitating the development of a simplified model suitable for controller design and analytical evaluation [15].

$$\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Cx + Du \\
D &= 0
\end{aligned}$$

where

$\phi = x_1 \rightarrow \dot{\phi} = \dot{x_1} \rightarrow \dot{\phi} = x_2$

$\theta = x_3 \rightarrow \dot{x_3} = x_4 \rightarrow \dot{\theta} = x_4$

$\psi = x_5 \rightarrow \dot{\psi} = x_6 \rightarrow \dot{x_5} = x_6$

$$A = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

$$B = \begin{vmatrix} 0 & 0 & 0 \\ \frac{1}{I_{xx}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{I_{yy}} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_{zz}} \end{vmatrix} \qquad (4)$$

$$C = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{vmatrix}$$

3. Fault Detection Using Luenberger Observer

In this study, a Luenberger observer is employed to estimate the system states and identify actuator faults simultaneously. By using the system inputs as observer inputs and producing estimated state variables as outputs, faults within the dynamic system can be effectively detected. Since the Luenberger observer is inherently designed for linear systems, it has been chosen as the method of choice in the present work. The conceptual block diagram of the proposed approach is shown in Figure 2.

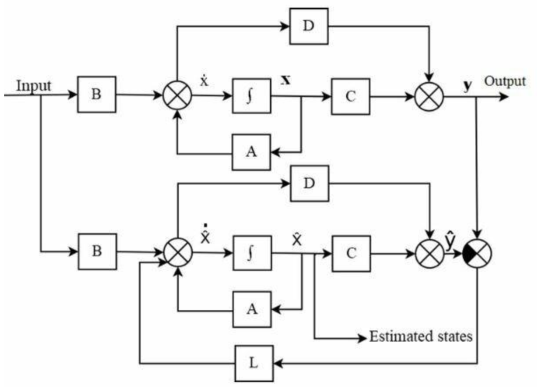

Figure 2. Block diagram of a linear observer

In this estimator, assume that the dynamical system is given in the usual state space representation as:

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (5)$$

The control system is designed using state feedback, $u(t) = -kx(t)$, under the assumption that $x(t)$ is available for measurement. In practice, however, the state $x(t)$ cannot be directly measured; only the system output, as defined in Eq. (6), is accessible.

$$y(t) = Cx(t) \qquad (6)$$

Here, $y(t)$ is referred to as the measurement variable.

If a dynamical system is considered in which the inputs are the measured output vector $y(t)$ and the control input vector $\boldsymbol{u}$, and the output is the estimated state vector $\hat{x}$, the corresponding state-space representation of this system can be expressed as Eq. (7).

$$\dot{\hat{x}}(t) = \hat{A}\,\hat{x}(t) + \hat{B}u(t) + Ly(t) \qquad (7)$$

Where $\hat{x}$ is an n-dimensional vector, and the matrices B, A, and L must be chosen such that the observation error of Eq. (8) tends to zero asymptotically. This observer design method is the Luenberger method.

By using the previous Equations, a dynamic Equation for the error can be obtained in the form of Eq. (8).

$$e(t) = x(t) - \hat{x}(t) \qquad (8)$$

$$\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t) \Rightarrow$$
$$\dot{e}(t) = Ax(t) + Bu(t) - \hat{A}\,[x(t) - e(t)] - \hat{B}u(t) - LCx(t) \Rightarrow \qquad (9)$$
$$\dot{e}(t) = \hat{A}\,e(t) + \left(A - LC - \hat{A}\right)x(t) + \left(B - \hat{B}\right)u(t)$$

If it is desired that the observation error independent of $x(t)$ and $u(t)$ asymptotically becomes zero, we must set the coefficients $x(t)$ and $u(t)$ in Eq. (9) Equal to zero and $\hat{A}$ corresponds to a stable dynamical system. Therefore:

$$\hat{A} = A - LC$$
$$\hat{B} = B \qquad (10)$$

Hence, the choice of $\hat{B}, \hat{A}$ and L is not arbitrary. Therefore, by substituting Eq. (10) into Eq. (9) and rewriting it as:

$$\dot{\hat{x}}(t) = (A - LC)\,\hat{x}(t) + Bu(t) + Ly(t)$$
$$\dot{\hat{x}}(t) = (A - LC)\,\hat{x}(t) + [\,B \;\; L\,] \begin{vmatrix} u(t) \\ y(t) \end{vmatrix} \qquad (11)$$

Eq. (11) describes the observer as the system state Equation. This Equation can be written as:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L\,[y(t) - C\hat{x}(t)] \qquad (12)$$

Negative system poles ensure rapid convergence to the steady state, which, in the present context, corresponds to obtaining the optimal state estimate. Conversely, poles with non-negative real parts—whether positive or zero—can cause pronounced oscillations or even prevent the system from reaching a steady state. The presence of negative poles accelerates the decay of the estimation error, thereby ensuring the stability of the system [16].

4. Simulation and analysis of results

In this section, to evaluate and validate the performance of the proposed approach based on the Luenberger method, a simulation was conducted on a quadrotor with the specifications listed in Table 1, using MATLAB/Simulink software.

Table 1. General specifications of the quadcopter

Parameter	Value	Unit	Symbol
Moment of inertia about x-axis	0.1676	$kg \cdot m^2$	I_{xx}
Moment of inertia about y-axis	0.1676	$kg \cdot m^2$	I_{yy}
Moment of inertia about z-axis	0.2974	$kg \cdot m^2$	I_{zz}

The error created here is applied to the actuator as a bias, as follows

$$U_{error} = 0.9 \times u + 0.01 \quad (13)$$

A fault is applied to the system during the time window of [5s, 20s].

The fault is introduced into the actuator during the time interval from $t = 5(\text{Sec})$ to $t = 20(Sec)$, under the assumption that it affects the roll actuators. In this context, the observer gain matrix L is determined as given in Eq. (14) and subsequently implemented within the Luenberger estimator to enable both system state estimation and fault detection.

$$L = \begin{bmatrix} 1.0229 & -0.000225 & -0.0001 \\ 0.2515 & 0.1131 & -0.1 \\ 0.2316 & 3.2771 & 0.5 \\ 1.1168 & 1.3886 & 0 \\ 1.2 & -1.8 & 0.3 \\ 2 & -1.5 & 0.52 \end{bmatrix} \quad (14)$$

Based on these values and the conducted simulation, the roll, pitch, and yaw angles, along with their estimated values, are shown in Figures 3 to 5, respectively.

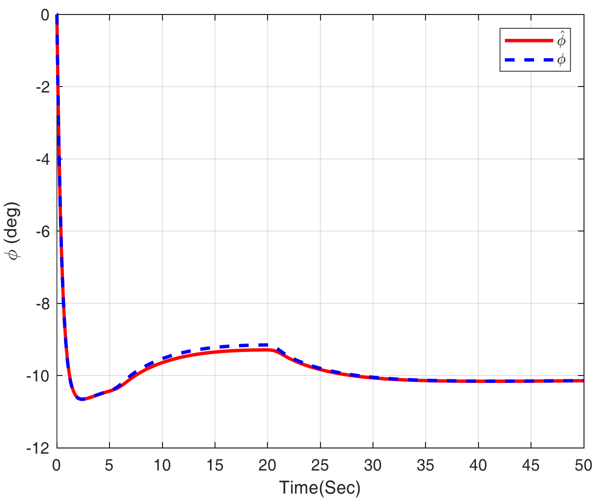

Figure 3. Roll angle of a quadrotor aircraft despite errors

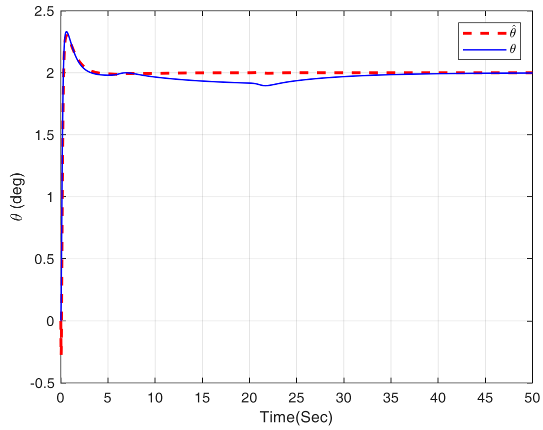

Figure 4. Pitch angle of a quadrotor

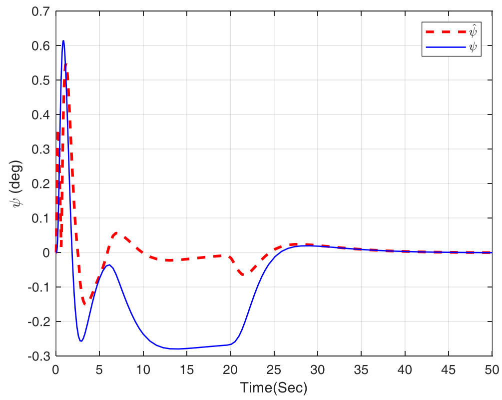

Figure 5. Yaw angle of the quadrotor

As observed, the actuator fault has introduced an error into the system. The angular velocities of the quadrotor are presented in Figures 6 to 8.

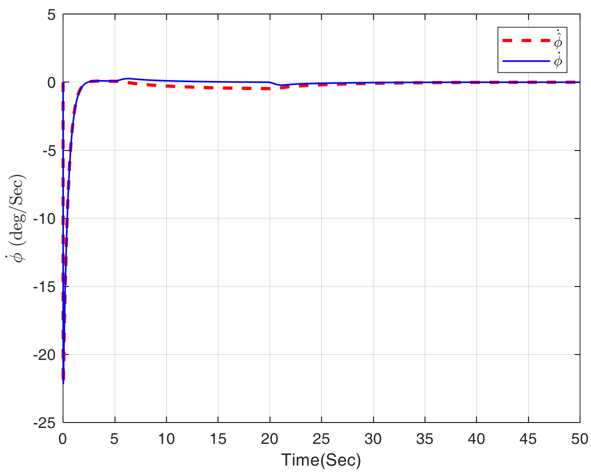

Figure 6. Angular velocity of X axis

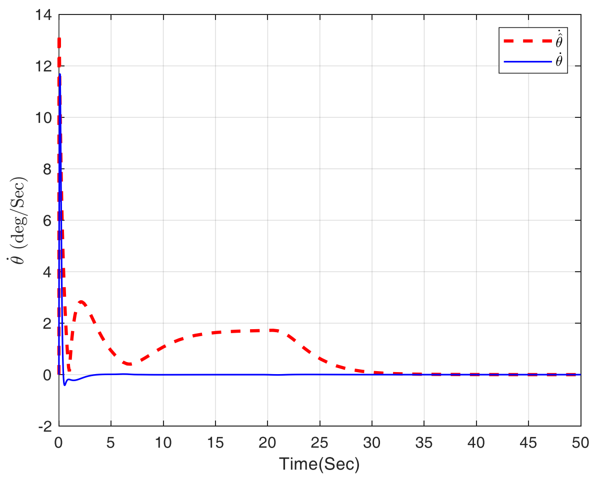

Figure 7. Angular velocity of Y axis

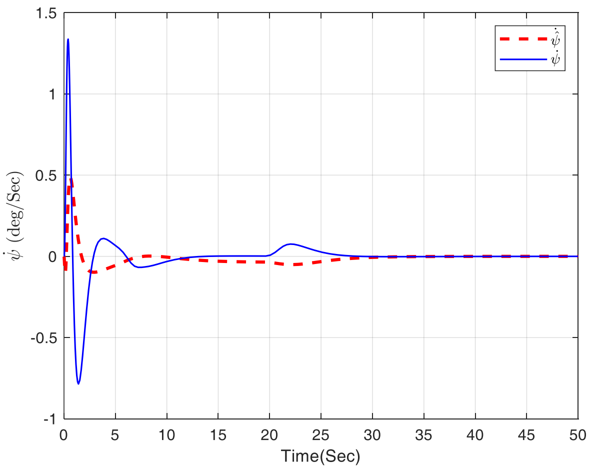

Figure 8. Angular velocity of Z Axis

The angular velocity also indicates the location of the error introduced in the actuator. In this analysis, the threshold limit is set to 0.05. Figures 9 and 10 present the roll and pitch estimation errors for two methods—the Luenberger estimator and the Kalman filter.

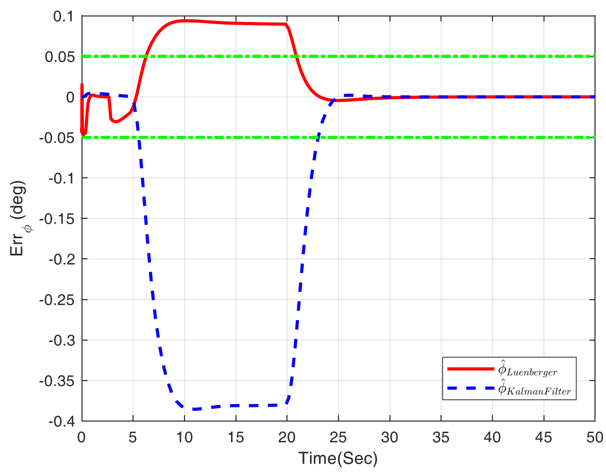

Figure 9. Roll angle error detection

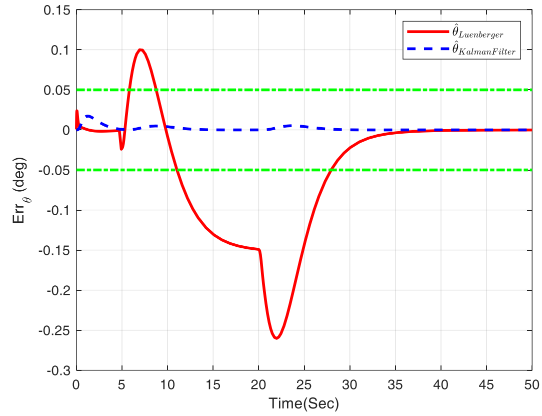

Figure 10. Pitch angle error detection

The figures above demonstrate that the Luenberger estimation method successfully detected the fault. However, as observed, the Kalman filter failed to identify any error in the pitch angle. Moreover, the fault detection time is significantly shorter for the Luenberger observer, which detects the fault at $t = 5.175(Sec)$ (0.175 after injection), compared to the Kalman observer, which detects it at $t = 5.536(Sec)$.

The mean, standard deviation (Std), and mean absolute error (MAE) of the estimation error are important quantitative measures for assessing an observer's or estimator's fault detection performance. These measures are listed in Table 2 for three time periods: before the fault, during the fault, and after the fault.

Table 2. Comparison of error criteria for quadcopters

Parameter	Error Angle	$t < 5(s)$	$5 < t < 20(s)$	$t > 20$
MAE	roll	0.0.0249	0.0804	0.0223
	pitch	0.0017	0.0156	0.0023
	yaw	0.117	0.3152	0.1479
Std	roll	0.0235	0.04	0.035
	pitch	0.0018	0.00189	0.0223
	yaw	0.1049	0.1947	0.1898
Mean	roll	−0.0246	0.0803	0.0207
	pitch	0.0016	−0.0025	−0.023
	yaw	0.1136	−0.3125	−0.147

5. Conclusion and Future Work

In this study, a Luenberger observer–based framework was employed to detect and isolate actuator faults within a quadrotor platform. By combining an accurate mathematical model with a rigorous analysis of the system's dynamic characteristics, the proposed methodology effectively identified and isolated both incipient and pronounced degradations in actuator performance, as well as anomalies in overall system operation.

Simulation results confirmed the effectiveness of the Luenberger observer algorithm, demonstrating high precision and rapid responsiveness in fault detection. The principal advantages of this approach include heightened sensitivity to early-stage faults, an intrinsic capability to reduce false alarms, and adaptability to a wide range of operating conditions. Furthermore, the findings indicate that employing this algorithm can significantly improve the stability and reliability of quadrotor systems, enabling their use in safety-critical missions such as search and rescue operations, high-resolution mapping, and aerial transportation.

Future work could involve implementing the proposed algorithms in a hardware-in-the-loop (HIL) environment and conducting validation experiments.

Conflict of Interests

No conflict of interest has been expressed by the authors.

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