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Original Research Article

A New Method Based on An Absorbing Markov Chain to Determine the Optimal Preventive Maintenance Policy

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Abstract

In this paper, a machine with five operational states is considered. This machine starts working in state one at 100% nominal capacity. The machine's performance decreases, and the machine enters states medium and bad, respectively. The operational process is modeled using a discrete Markov chain, a transition probability matrix is obtained, and the total cost of the system is evaluated considering costs, such as lost production cost, PM cost, as well as the operational costs of the machine entering each of the system states. The objective function is cost minimization. In this model, for different policies, the average cost of the production process is calculated, and the policy with the lowest production process cost is selected as the optimal maintenance and repair policy. Unlike many previous approaches that primarily rely on theoretical analysis or simplified modeling, the proposed framework employs a discrete-time absorbing Markov process to facilitate cost-based decision-making in maintenance and repair operations. By considering various functional states of equipment, transition probabilities between these states, and the associated costs of maintenance actions, the model offers optimal and implementable policies for short-term maintenance management. Key advantages of this model include its ability to simulate equipment behavior realistically, provide more accurate estimates of maintenance costs, and analyze practical strategies to minimize unplanned downtime and enhance operational efficiency.

Keywords: Discrete Markov Chain; Maintenance and Repair; Optimal Policy; Cost Minimization.

1. Introduction

One of the main problems in industries is the sudden failure of machinery during production, which disrupts the production system and increases costs incurred by the system, such as production line stoppage costs, machinery repair or replacement costs, customer delivery delay costs, and even sales costs. One of the most essential methods used in industrialized countries is planning a set of systematic instructions and processes to prevent sudden machinery failures [1]. The method of increasing machine life is known as a maintenance policy. When shafts, belts, and other components wear out, the failure of any of these components can cause the machine and production line to stop [2]. To ensure equipment reliability, implementing an optimal maintenance policy is crucial, as incorrect maintenance policies can lead to increased costs due to lost production [3]. Given the sudden failures of machinery in production systems, determining an optimal maintenance policy is crucial for ensuring optimal production. One of the primary objectives of maintenance policies is to minimize unplanned equipment downtime. Consequently, when a machine is under control, it leads to increased productivity [4]. In this regard, various models exist for determining optimal maintenance policy. Kumar et al. [4] proposed a probabilistic model for predicting optimal availability using the Markov method and differential equations. This model utilizes the Grey Wolf Optimizer, Whale Optimization Algorithm, and Lion Swarm Algorithm to predict optimal profit. Yang et al. [5] proposed a mixed-integer linear programming

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formulation to optimize the integrated long-term maintenance schedule. They used an algorithm that combines Benders decomposition and Lagrangian relaxation to improve calculation speed.

Thomasevich and Asgari [6] proposed a model to find the optimal time for preventive maintenance for a machine using Markov model relationships, which includes eight states: three operational states, two failure states, and three preventive maintenance states. Amari et al. [7] considered equipment with degrading properties in six states. Unlike Thomasevich and Asgari's [6] study, where two actions are feasible in each state, this study allows for three actions in each state, each with specified rates: performing PM, performing CM, or continuing operation. Finally, using a continuous-time Markov process, the optimal action in each state is determined to maximize equipment reliability. In another study, Anderson et al. [8] proposed a model that defines the optimal replacement time for a multi-component system using a continuous-time Markov model and dynamic programming. Most previous studies have considered a continuous-time Markov chain because failure times are easily obtained using these models. The state of a system can be modeled as a discrete-time Markov chain because absorbing states exist in real-world problems. Therefore, this research considers a production process that includes five states, two of which are absorbing. Using an absorbing Markov process, a mathematical model is developed to analyze the maintenance policy for the production process. The optimal maintenance policy is determined using discrete-time Markov chain equations such that the objective cost function is minimized.

2. Literature Review

Recently, many studies have been conducted in the field of optimizing machinery maintenance systems. Al-Husseini et al. [9] determined the optimal time for performing PM activities in public vehicles. In this model, the optimal time for maintenance was determined using a mixed-integer programming approach. Fakharooti et al. [10] determined optimal maintenance strategies for railway networks using non-linear integer programming. Kew et al. [11] presented a model that, through mathematical modeling and consideration of probabilistic orders, production fluctuations, and inter-machine dependencies, determines the optimal maintenance policy to minimize failure, maintenance, and repair costs. Zhang et al. [12] proposed a mixed-integer linear programming model for optimizing the maintenance schedule of industrial generators. [13] proposed a model for optimizing the maintenance of natural gas transmission pipelines, considering external corrosion factors. Fallahnejad et al. [14] determined the optimal time for equipment inspection by combining Bayesian inference and statistical reliability model-based preventive maintenance methods. Alle et al. [15] proposed a method for optimizing the maintenance schedule of wind turbines, using a simulation-optimization method. The proposed model, the Ant Colony algorithm, is used to optimize the routing of maintenance activities. Fallahnejad and Niaki [16] presented a Markov model that minimizes system cost and determines the optimal maintenance policy for a machine comprising two states. Anderson et al. [17] determined the optimal time model for replacing parts of a multi-component wear system following a multivariate gamma distribution using dynamic programming. In this model, the optimal replacement time for system components is determined. Kamal et al. [18] optimized the PM frequency for multiple machines using a genetic algorithm to represent decision variables for repair, replacement, and inspection in terms of reliability, availability, and maintenance cost. Lin et al. [19] employed an estimated failure time distribution to construct a cost function, selecting the lowest expected long-term cost rate as the optimization objective to determine the preventive maintenance period. Shi et al. [20] developed an optimal maintenance strategy for a complex periodic inspection system using an imperfect maintenance model, with the objective of minimizing the cost of the inspection and maintenance process. Sarkar and Fayezi [21] developed a maintenance cost model for offshore wind turbine components following a multi-level opportunistic PM strategy that considers replacement and preventive maintenance. Mahmoud et al. [22] employed the Analytic Hierarchy Process to develop a model for selecting the most suitable maintenance strategy for cement plants. Al-Jubouri et al. [23] used a robust optimization framework to propose a model for optimizing the joint selective maintenance problem and maintenance worker allocation when the quality of maintenance actions is uncertain. Sajini et al. [24] proposed a model for optimizing offshore power supply accessibility.

By reviewing the literature on maintenance and repair optimization, particularly studies employing Markov chain models, it was observed that most existing approaches assume a continuous system state and rely on continuous-time Markov processes for problem formulation and optimization. Moreover, these models typically exclude the presence of absorbing states. However, in real-world applications, system states are inherently discrete, and absorbing states, such as a failure state, exist. To address this gap, the present study utilizes discrete-time Markov chain relationships to determine the optimal maintenance and repair policy that minimizes the cost objective function that denotes the average cost of the production process. Through the application of absorbing Markov chain equations and the transition probability matrix, key parameters of the objective function are derived, including the expected number of visits to transient states before absorption, transition probabilities among transient states (excluding absorbing states), the likelihood of absorption from each transient state, and the expected time to system failure. These parameters are then used to calculate the average cost of the production process, allowing for the selection of the optimal policy.

3. Problem Statement

The problem considered in this research involves a machine in operation with two possible types of failures: deterioration and sudden failure.

At any time, the machine is in one of the following states:

Mode 1: Good machine performance and operation at 100٪ capacity.

Mode 2: Medium machine performance and operation at 80٪ capacity.

Mode 3: Low machine performance and operating at 50٪ capacity.

Mode 4: Deterioration failure and machine stop.

Mode 5: Sudden breakdown and machine stop.

At the end of each stage, the machine's condition is determined by performing an inspection.

Initially, the machine starts operating from state 1 (100% capacity), and in the next stage, it enters state 2 or 5 with specific probabilities, or remains in its current state with particular probabilities. If the machine enters state 2, two decision-making policies can be applied:

Policy 1: Continuation of machine operation with medium performance.

Policy 2: Machine repair.

Suppose policy 1 is selected and the decision is made to continue the operation of the machine with medium performance, in the next step. In that case, the machine will enter one of the states 3 or 5 with specific probabilities, or it will remain in its current state with a specific probability.

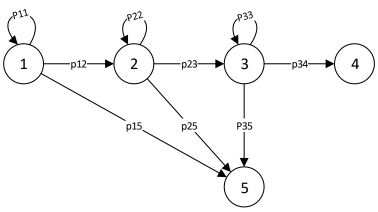

Figure 1. State diagram for the policy of continuing machine operation in State 2

According to the above figure, the maintenance strategy does not involve any repair or replacement actions throughout the operational steps.

Suppose policy 2 is selected and a repair decision is made by imposing the cost of repair on the system. In that case, the machine will enter one of the states 1, 3, or 5 with a specific probability, or it will remain in its current state with a specific probability. In this case, the probability of transition to state 1 is greater than the other probabilities.

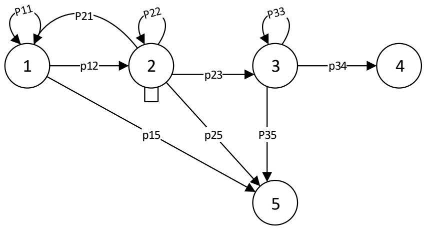

Figure 2. State diagram for the machine repair policy in State 2

According to the above figure, in state 2, the repair action is performed and the machine returns to state 1 with a specified probability.

In the policy of continuing the machine operation, the machine enters state 3, and then two decisions can be applied in state 3:

Policy 1: Continuation of machine operation in state 3.

Policy 2: Machine repair and replacement in state 3.

Suppose policy 1 is selected and the decision is made to continue operating the machine with poor performance. In that case, the machine enters one of the states 4 or 5 with specific probabilities or remains in its current state with a specific probability in the next step.

Suppose policy 2 is selected and a decision is made to repair and replace the machine by imposing the cost of repair and replacement on the system. In that case, the machine enters one of the states 1, 2, 4, or 5 with a specific probability, or remains in its current state with a specific probability. However, in this case, the probability of entering state 1 is greater than the probabilities of entering the other states.

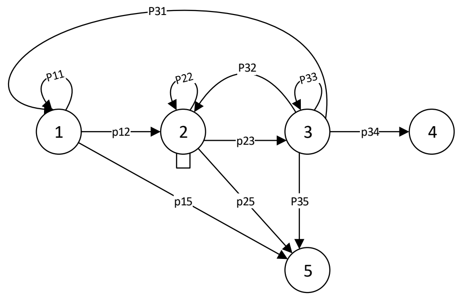

Figure 3. State diagram for machine replacement in state 3

According to the above figure, in state 3, the replacement action is performed and the machine returns to state 1 or 2 with a certain probability.

When the machine is in one of the states 4 or 5, it is impossible to return it to other states. These states are the absorbing states of the system, and a new machine must replace the machine.

This research aims to select the optimal policy in cases 2 and 3 to reduce the average cost of the production process. Thus, the objective is to determine the time during which preventive maintenance can be applied in states 2 and 3 or both states. It is assumed that the repair and replacement decision is imperfect, and it improves the performance of the machine and transitions its condition to better states. Thus, there are four decision-making:

Policy 1: Continuation of machine operation with medium performance in state 2 and operation with poor performance in state 3. (Figure 1)

Policy 2: Machine repair was performed in state 2, and operation was continued with poor performance in state 3. (Figure 2)

Policy 3: Continue to operate the machine with medium performance in state 2 and repair or replace the machine in the state 3. (Figure 3)

Policy 4: Repair the machine in state 2 and repair or replace the machine in the state 3. (Figure 4)

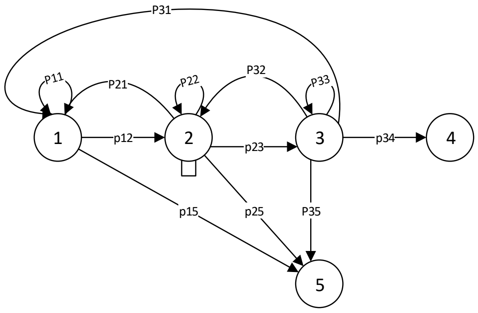

Figure 4. State diagram for the policy of repairing the machine in state 2 and repair or replacement parts in state 3

According to the above figure, in the state 2, the repair action is performed and the machine returns to state 1 and in the state 3, the replacement action is performed and the machine returns to state 1 or 2 with a specified probability.

The Markov mathematical model of the maintenance problem can be formulated by taking into account the transition probabilities of changing the system states and the costs imposed on the system by each of these policies to select the optimal policy in such a way that the average cost of the production process can be minimized.

The notations used in this paper are as follows:

P_{ij}= Transition probability from state i to state j.

C_i= Expected cost in state i.

F(C) = objective function.

T = Time to failure vector.

F_{ij}= Probability of absorption from transient state i to absorbing state j.

P_i = Transition probability matrix of system states at policy i.

C_{re} = the cost of repair.

C_{rp} = the cost of replacement.

Q = Transition probability matrix among the transient states.

R = Elements related to the rows of transient states and columns of absorbing states.

M = the number of transient states in the system.

N = the fundamental matrix for P, which denotes the expected number of states before absorption.

N_{ii} = The expected number of transitions to stage i

If policy 1 is chosen, the calculations of different probabilities are as follows:

The matrix P, which is the probability transition matrix of the system states, is as follows:

$$P = \begin{bmatrix} P_{11} & P_{12} & 0 & 0 & P_{15} \\ 0 & P_{22} & P_{23} & 0 & P_{25} \\ 0 & 0 & P_{33} & P_{34} & P_{35} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The transition probability matrix Q among the transient states of the system is obtained from the rows and columns of the non-absorbing states of the system in matrix P as follows:

$$Q = \begin{bmatrix} P_{11} & P_{12} & 0 \\ 0 & P_{22} & P_{23} \\ 0 & 0 & P_{33} \end{bmatrix} \quad (2)$$

The matrix N (The fundamental Matrix for P), denoting the expected number of transitions to each state before being absorbed, is determined as follows:

$$N = [I - Q]^{-1} \quad (3)$$

Parameter T_1 is the expected number of steps before the chain is absorbed (Time to failure), and it is obtained as follows:

$$T = N * 1 \quad (4)$$

T_1 is the first element of vector T. F_{ij} denotes the probability of absorption from transient state i to absorbing state j, which is obtained by the following equation:

$$F = N * R \quad (5)$$

The objective function of the problem is obtained as follows:

$$F(C) = \frac{Replacement\ cost + Repair\ cost + Total\ failure\ cost + expeted\ operation\ cost}{Time\ to\ failure} = \frac{0+0+C_4F_4+C_5F_5+N_{11}C_1+N_{22}C_2+N_{33}C_3}{T} \quad (6)$$

Here, $C_4F_4 + C_5F_5$ is the expected failure cost, $N_{11}C_1 + N_{22}C_2 + N_{33}C_3$ is the expected operation cost, and T is the time to failure. The expected number of transitions to state 1 is multiplied by the operating cost of state 1 in terms of $N_{11}C_1$.

If policy 2 is selected, the average cost is obtained as follows,

$$F(C) = \frac{Replacement\ cost + Repair\ cost + Total\ failure\ cost + expeted\ operation\ cost}{Time\ to\ failure} = \frac{0+C_{re}+C_4F_4+C_5F_5+N_{11}C_1+N_{22}C_2+N_{33}C_3}{T} \quad (7)$$

Here, C_{re} is the cost of repair, $C_4F_4 + C_5F_5$ is the expected failure cost, $N_{11}C_1 + N_{22}C_2 + N_{33}C_3$ is the expected operation cost, and T is the time to failure.

If policy 3 is selected, the average cost in the third policy is obtained as follows.

$$\mathrm{F(C)} = \frac{Replacement\ cost + Repair\ cost + Total\ failure\ cost + expeted\ operation\ cost}{Time\ to\ failure} = \frac{C_{rp} + 0 + C_4F_C + C_5F_5 + N_{11}C_1 + N_{22}C_2 + N_{33}C_3}{T} \quad (8)$$

Here, C_{rp} is the cost of replacement, $C_4F_4 + C_5F_5$ is the expected failure cost, $N_{11}C_1 + N_{22}C_2 + N_{33}C_3$ is the expected operation cost, and T is the time to failure.

If policy 4 is selected, the average cost in the third policy is obtained as follows:

$$F(\mathrm{C}) = \frac{Replacement\ cost + Repair\ cost + Total\ failure\ cost + expeted\ operation\ cost}{Time\ to\ failure} = \frac{C_{rp} + C_{re} + C_4F_4 + C_5F_5 + N_{11}C_1 + N_{22}C_2 + N_{33}C_3}{T} \quad (9)$$

Here, C_{rp} is the cost of replacement, C_{re} is the cost of repair, $C_4F_4 + C_5F_5$ is the expected failure cost, $N_{11}C_1 + N_{22}C_2 + N_{33}C_3$ is the expected operation cost, and T is the time to failure.

The average cost of each policy can now be evaluated, and the one with the minimum average cost is optimal.

4. Case study

A Bag Filter, one of the main parts of the steel industry, with the following parameters, is considered a case study:

$$c = [100 \quad 130 \quad 150 \quad 1000 \quad 1500]$$

$$P_1 = \begin{bmatrix} 0.85 & 0.1 & 0 & 0 & 0.05 \\ 0 & 0.8 & 0.1 & 0 & 0.1 \\ 0 & 0 & 0.8 & 0.1 & 0.1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0.85 & 0.1 & 0 & 0 & 0.05 \\ 0.8 & 0.1 & 0.05 & 0 & 0.05 \\ 0 & 0 & 0.8 & 0.1 & 0.1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0.85 & 0.1 & 0 & 0 & 0.05 \\ 0 & 0.8 & 0.1 & 0 & 0.1 \\ 0.5 & 0.3 & 0.1 & 0.05 & 0.05 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 0.85 & 0.1 & 0 & 0 & 0.05 \\ 0.8 & 0.1 & 0.05 & 0 & 0.05 \\ 0.5 & 0.3 & 0.1 & 0.05 & 0.05 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_{re} = 500(\$)$

$C_{rp} = 1000(\$)$

The average cost of the production process is calculated in Table 1, based on which the optimal policy is selected.

Table 1. Average cost of the production process in each production cycle for different policies

Policy Type	F(c)
Policy1	302.6$
Policy2	254.4$
Policy3	325.6$
Policy4	269.4$

According to the results obtained, it is determined that Maintenance Policy 2, with an average process cost of \$254.40, is the best policy. Therefore, repairing the machine in state 2 and continuing the operation in state 3 is the optimal policy.

5. Comparison study

In this section, the proposed method is compared with the method of Kargar et al. [28]. The operating cost of the proposed system is calculated by multiplying the number of transitions to each state by the cost associated with each state. Kargar et al. [28] proposed evaluating the average cost by multiplying the limiting probabilities of each state by the cost of each state.

The objective function in Kargar et al. [28] for the policy 1 is obtained as follows:

$$\mathrm{F(C)} = \frac{Replacement\ cost + Repair\ cost + Total\ failure\ cost}{Time\ to\ failure} + expeted\ operation\ cost = \frac{C_{rp} + C_{re} + C_4F_C + C_5F_5}{T} + \pi_1 C_1 + \pi_2 C_2 + \pi_3 C_3 \quad (10)$$

The probability π_i that denotes the limiting probability of the state i is obtained using equilibrium equations as follows,

$$\pi * S = \pi \quad (11)$$

$$\sum_{i=1}^{M} \pi_i = 1 \quad (12)$$

The matrix S represents the probability of transition between the transient states of the system. The matrix S and the objective function for policy 1 are obtained as follows:

$$S = \begin{bmatrix} P_{11} + P_{15} & P_{12} & 0 \\ P_{25} & P_{22} & P_{23} \\ P_{35} + P_{347} & 0 & P_{33} \end{bmatrix} = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0.2 & 0 & 0.8 \end{bmatrix} \quad (13)$$

$$\mathrm{F(C)} = \frac{C_{rp}+C_{re}+C_4F_C+C_5F_5}{T} + \pi_1 C_1 + \pi_2 C_2 + \pi_3 C_3 = \frac{0+0+(1000*0.16)+(1500*0.84)}{11.5} + (0.57*100) + (0.29*130) + (0.14*150) = 239.17\$ \quad (14)$$

The results of the proposed method are compared with the results of Kargar et al. [28]. The results are denoted in Table 2.

Table 2. The values of the objective functions for different policies based on the method of Kargar et al. [28]

Policy Type	F(c)
Policy1	239.17$
Policy2	208.75$
Policy3	289.04$
Policy4	261.26$

According to the results presented in Table 2, it is evident that the optimal policy identified by the method proposed by Kargar et al. [28] is also Policy 2. The proposed approach evaluates the average cost at each decision stage, which makes its outcomes potentially more applicable and interpretable in practical settings compared to the model introduced by Kargar et al. [28]. Since repaired decisions often involve short-term actions, evaluating the cost of each state based on the number of transitions among stages is more adaptable with the policies in the short horizon of decision-making. In the latter, the average cost is derived from limiting probabilities, representing the expected cost over an infinite time horizon within a Markov decision process. However, in many real-world scenarios, decision-making occurs within a finite time frame. Therefore, assessing the average cost per stage provides more practical insights and can lead to more effective and sensitive policy selection. This stage-wise evaluation not only enhances the interpretability of the results but also aligns more closely with the operational constraints and time-sensitive nature of real-world decision-making environments. It should be mentioned that a similar approach is chosen in Khasawneh et al. [29] in a process setting problem.

6. Conclusions

Given that maintenance and repair costs contribute a significant share to the product's total costs, it is necessary to design a method for selecting an optimal maintenance and repair policy. In this paper, the optimal maintenance policy is determined by using discrete Markov chain relationships and calculating the average cost of the production process. The average cost of the production process is the sum of expected breakdown and operation costs. This research aimed to determine optimal maintenance and repair policies. In future research, assumptions such as considering multiple machines instead of a single machine and the impact of a machine breakdown on the performance of other machines can be incorporated into the problem. The optimal maintenance and repair policy can be calculated under different conditions. In addition to minimizing the cost, maximizing the machine's reliability can be considered in the problem. The optimal maintenance and repair policy can be calculated for a multi-objective problem.

Conflict of Interests

No conflict of interest has been expressed by the authors.

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