

Development of a Model to Evaluate the Efficiency of Maintenance Units for Automated Guided Vehicles in Industry 4.0 Using an Integrated Approach Based on Data Envelopment Analysis and Game Theory

Maziyar Massahi , Hamid Tohidi *  and Seyedehfatemeh Golrizgashti 

Department of Industrial Engineering, ST.C., Islamic Azad University, Tehran, Iran

* H.tohidi@iau.ac.ir

Abstract

With the advent of the Fourth Industrial Revolution, the use of specialized tools and technologies in industrial systems has significantly expanded. Among these, Automated Guided Vehicles (AGVs) have emerged as a vital component in factories aligned with Industry 4.0 standards. However, like any other equipment, AGVs require systematic and ongoing maintenance to ensure optimal performance. This study aims to evaluate the efficiency of AGV maintenance units through an integrated model that combines Data Envelopment Analysis (DEA) and Game Theory. The model is applied under conditions that assess the impact of various maintenance strategies—corrective, preventive, and predictive—alongside the spatial configuration of maintenance and depot sites. The incorporation of Game Theory accounts for the cooperative effects among decision-making units (DMUs), enhancing the realism and analytical depth of the evaluation. The integration of these strategies with Game Theory is novel, as previous studies have generally explored predictive maintenance in isolation, without examining the combined effects with other maintenance strategies.

The results from the integrated DEA–Game Theory analysis indicate that the proposed network structure provides valuable insights into the sources of inefficiency across DMUs. This allows for the identification of optimal maintenance strategies and the most effective spatial arrangements for maintenance and depot facilities. A comparative analysis with the classical DEA model reveals that, out of three efficient DMUs in the traditional framework, only one remains efficient under the integrated model, while the other two show inefficiencies across at least three sub-networks.

Ultimately, the findings suggest that adopting a predictive maintenance approach—alongside corrective and preventive strategies—yields superior performance. Moreover, the co-location of AGV maintenance and depot facilities with existing production machinery maintenance sites is identified as the most efficient configuration. This novel combination of maintenance strategies, evaluated through the integrated DEA–Game Theory model, offers significant practical value for optimizing maintenance operations in the context of smart manufacturing.

Keywords: Maintenance Efficiency Evaluation; Data Envelopment Analysis; Automated guided vehicles; Industry 4.0; Game Theory; Network DEA Model; Hybrid Decision-Making Approach.

1. Introduction

Automated Guided Vehicles (AGVs) are extensively employed across a wide range of industries and production, particularly in sectors where the internal transportation of materials, parts, and equipment is essential. Their widespread adoption is attributed to their high flexibility, precision, and operational efficiency. As

AGV fleets continue to expand in both scale and complexity, challenges related to system reliability and maintenance become increasingly critical [1].

Functionally, AGV systems operate as computer-controlled material handling networks, enabling seamless operations across production lines, warehouses, terminals, and external logistics environments. Due to their adaptability and autonomous capabilities, AGVs

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have proven to be highly effective in dynamic industrial settings [2].

As these systems become more sophisticated, enhancing their operational efficiency and minimizing maintenance-related costs become strategic priorities. Achieving these objectives requires an in-depth analysis of AGV design and control mechanisms, as well as a comprehensive understanding of flow path layouts and workstation configurations. Additionally, operational factors such as vehicle speed, braking distance, reliability requirements, and mean time to failure (MTTF) must be thoroughly analyzed to ensure safety and efficiency [3].

With the emergence of modern industrial paradigms, issues related to the maintenance and dependability of AGVs have gained increasing prominence. Despite this, limited research has been conducted to specifically address the challenges associated with AGV maintenance. Currently, engineering practices primarily rely on two conventional strategies—corrective and preventive maintenance. However, over recent decades, considerable research has focused on optimizing these strategies across various industrial contexts [4].

In response to this evolving landscape, the present study investigates the combined effect of maintenance location and strategy on the overall efficiency of AGV maintenance operations. The proposed model integrates multiple analytical approaches, including location analysis, multi-criteria decision-making (MCDM), and Game Theory.

The central research question is as follows:

How can a model be developed to effectively assess the efficiency of AGV maintenance units using an integrated framework based on Data Envelopment Analysis and Game Theory?

To address this question, the study begins with a review of relevant literature, introduces the proposed hybrid model, presents the analytical results, and concludes with key insights and recommendations.

2. Literature Review

This section provides a review of existing studies on AGV maintenance, followed by an examination of research employing Data Envelopment Analysis (DEA) techniques.

In [5], the authors implemented a system comprising three AGVs and proposed a scientific approach to optimize layout design, operational processes, and maintenance strategies in multi-AGV systems. Several studies have examined various aspects of AGV system optimization. For instance, [2] addressed the problem of material transportation allocation by modeling it as a multi-cargo task assignment issue. In line with improving system reliability, [4] proposed an effective maintenance strategy for multi-AGV systems based on genetic algorithms. Furthermore, [6] focused on optimizing job transfers and control mechanisms using artificial intelligence techniques, particularly in the context of

AGV routing. Complementing these efforts, [7] developed a comprehensive scheduling framework for AGV task execution and conducted an in-depth analysis of operational processes.

In recent years, topics related to improving the performance of complex systems through the use of emerging technologies have garnered widespread attention. For instance, in a different but related field concerning distributed and fault-sensitive systems, [8] has conducted a comprehensive review, demonstrating that systematic risk and safety management in aerospace operations can play a crucial role in ensuring safe performance by utilizing standard risk assessment methods. Their findings emphasize that in complex systems such as automated and robotic networks, a detailed focus on risk and safety can be impactful and beneficial in the design of mechanisms for error prediction and prevention.

In addition, [9] introduced a set of simulation modules within Siemens factory simulation software that automatically generate models of robotic assembly systems integrated with AGVs, supporting both maintenance and production planning. Building on this, [1] presented an optimized maintenance strategy for a complex multi-AGV system consisting of several vehicles operating along defined paths and executing designated tasks.

In the context of port logistics, [10] explored advanced technologies for optimizing port operations and automating logistics processes, contributing significantly to enhancing the strategic role of ports in the global maritime industry. Complementarily, [11] assessed AGV transportation systems and evaluated their reliability, while [12] emphasized the importance of synchronized scheduling of loading and unloading operations—both in time and space—to enhance AGV efficiency at terminals.

Organizational structure was addressed by [13], who noted that most enterprises operate within hierarchical systems, thereby requiring suitable models for evaluating networked efficiency. DEA has been widely recognized in this context as an effective method for assessing the relative efficiency of decision-making units (DMUs) that convert multiple inputs into multiple outputs. Expanding on this, [14] emphasized that the efficiency of transportation companies is crucial not only for individual firms but also for broader economic systems. Their study introduced an integrated model to evaluate the efficiency of representative transport companies over eight years.

Further developments include [15], who investigated scheduling challenges for AGVs under battery constraints, and [16], who focused on measuring overall efficiency in network DEA models, particularly for two-stage DMUs operating with shared resources—a domain that has seen notable methodological advancements. Finally, [17] assessed the efficiency of maintenance operations for water infrastructure using a hybrid method combining fuzzy inference systems with slack-based DEA. Similarly, [18] evaluated efficiency

based on novel performance metrics in the maintenance of comprehensive power generators using a combination of DEA and the fuzzy COPRAS method.

A comprehensive review of the literature highlights a significant research gap in the evaluation of maintenance efficiency for Automated Guided Vehicles (AGVs). This study seeks to address this gap. To date, no substantial domestic research has been conducted in this area, and the existing international studies are primarily limited to a series of works by Yan between 2017 and 2021. Nevertheless, these efforts fall short of fully bridging the gap, as they do not adequately account for Industry 4.0 environments or incorporate predictive maintenance strategies.

The review of prior studies reveals that most research on AGVs has primarily focused on aspects such as transportation task optimization [19], mission reliability modeling [3], or general maintenance strategy design [4]. While each of these contributions has enriched the field in meaningful ways, several structural limitations can be identified:

- Many studies assess performance only at a high system level and overlook the internal networked structure of maintenance processes;
- The use of simple DEA models often fails to capture inefficiencies rooted within mechanical, electrical, and operational subsystems;
- Most importantly, predictive maintenance—a critical component of Industry 4.0—has either been neglected or addressed only superficially.

Although some recent research has explored DEA integrated with Game Theory (e.g., [13], [14],[20]), their application domains have largely focused on sectors outside of AGV systems, such as finance, infrastructure, or general organizational efficiency. Recent methodological enhancements in DEA literature also warrant attention. For instance, [21] introduced a cross-evaluation-based Malmquist Productivity Index, addressing potential overestimations in traditional self-evaluated MPI measures by incorporating peer-based efficiency assessments. Their model offers a more robust and comparable productivity framework, which is particularly relevant for evaluating dynamic operational environments. Furthermore, [22] developed a formal statistical framework to test for heterogeneity in DEA models. Their approach is grounded in microeconomic production theory and helps uncover structural differences among decision-making units (DMUs), a critical step in ensuring model validity across diverse technological environments. Complementing these theoretical advancements, [23] applied a novel picture fuzzy BCC DEA model for evaluating hospital performance. Their implementation demonstrates the practical applicability of fuzzy DEA frameworks in complex service systems where data imprecision and ambiguity are prevalent.

As presented in Appendix, a detailed comparison of prior studies' objectives, methods, application domains,

and contributions further highlights these gaps. This comparison confirms that the present study is among the first to apply a hybrid DEA–Game Theory model specifically for evaluating the efficiency of AGV maintenance units under a multi-strategy framework that includes corrective, preventive, and predictive maintenance.

Additionally, by incorporating spatial configurations of maintenance and depot sites and conducting sub-network sensitivity analysis, this study introduces a flexible and adaptable framework for strategic decision-making. The model not only identifies the sources of inefficiency more precisely but also serves as a practical decision-support tool for smart manufacturing environments aligned with Industry 4.0 principles.

Consequently, the maintenance of AGVs represents a relatively new and underexplored domain that merits further academic investigation. As previously noted, this study concentrates on evaluating the reliability of multi-AGV systems. In particular, it focuses on the operational efficiency of maintenance units, given their critical influence on overall system performance. The study also analyzes key factors that affect this efficiency, including maintenance policies and the geographic location of maintenance facilities.

In the subsequent sections (Sections 3 and beyond), the study outlines the structure of the production environment, defines the mathematical model integrating DEA and cooperative Game Theory, and presents a detailed sub-network-based evaluation of AGV maintenance performance. Section 4 provides the efficiency results of the proposed model and compares them with classical DEA, while Section 5 discusses the managerial implications and strategic insights derived from the analysis. Finally, Section 6 concludes the study with key contributions, limitations, and directions for future research.

3. Production Environment and Modeling

3.1 Production Environment

The primary dataset for this study was obtained through pilot implementation, which aimed to evaluate the operational performance of AGVs under different configurations. To ensure the validity of the data, the experimental duration for each scenario was set to 10 days. This period was deemed sufficient to capture a variety of system behaviors. The results obtained from the 10-day simulations were then extrapolated for 6 months using a Python-based simulation model. This allowed for a more comprehensive view of the system's performance over a longer time frame, providing robust insights into the operational dynamics.

In this study, three potential configurations for AGV maintenance and depot sites were identified, considering operational costs and space usage. These sites can either

be co-located with the AGV charging depot or positioned independently. Furthermore, the maintenance site can either share space with the maintenance facilities for production equipment or be set up separately. The three resulting scenarios are:

- Scenario A: The AGV maintenance and depot sites are separate from the production equipment maintenance facility but are both located adjacent to the production line.
- Scenario B: The AGV maintenance site is shared with the production equipment maintenance facility, while the depot is situated away from the production line.
- Scenario C: The AGV maintenance site is shared with the production equipment maintenance facility, and the depot is located adjacent to the production line.

Regarding AGV maintenance strategies, three tiers were considered:

- Strategy A: Corrective maintenance only.
- Strategy B: Corrective + Preventive maintenance.
- Strategy C: Corrective + Preventive + Predictive maintenance.

By combining the three site configurations with the three maintenance strategies, a total of nine distinct scenarios (3 locations $\times$ 3 strategies) were generated. These scenarios served as the input for the integrated DEA and Game Theory model, as detailed in Table 1.

Additionally, it should be noted that the input parameters, including operational costs, were estimated based on available data and assumptions. The simulation considered various factors such as task completion times, buffer capacities, and distances between workstations, with a random task assignment process to simulate real operational dynamics.

3.2 Mathematical Model and Network Structure

This section introduces the integrated analytical framework, which combines Data Envelopment Analysis (DEA) with Game Theory. The model is formally described using mathematical expressions.

Many existing studies have assessed the efficiency of maintenance units without considering their internal network structure. However, in practical applications, especially when evaluating systems with varied maintenance strategies and reliability requirements, it is essential to incorporate the internal network configuration. This includes modelling the relationships between intermediate performance metrics and the sequential stages of the maintenance process.

A simplistic single-stage model is inadequate, as it fails to capture the full complexity and operational dynamics of maintenance units. Therefore, this research proposes a network-based structure that reflects the multifaceted nature of AGV maintenance operations. This approach also models the interaction between the decision-making units (DMUs) and their internal processes, where the DMUs represent key maintenance functions. The relationship between these

stages and their interdependencies is essential for ensuring optimal efficiency.

In general, the internal structure of the Decision-Making Units (DMUs), which represent the AGV maintenance processes responsible for handling materials and components along production lines, is depicted in Figure 1. This structure includes:

- The maintenance process
- The influence of the physical location of maintenance and depot units
- Personnel cost
- This includes the performance of AGV's parts, as defined in [3].
- The overall performance of the AGVs.

Table 1 presents the inputs and outputs associated with each component outlined in the Figure 1.

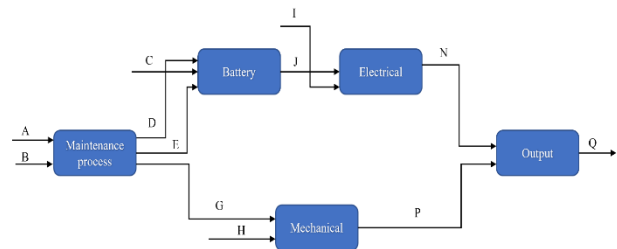


Figure 1. DEA Network Structure

Table 1. Inputs and Outputs

Symbol	Description
A	Total labor costs of maintenance technicians
B	Construction costs for the maintenance and depot facilities
C	Total cost of spare parts for battery-related maintenance
D	Total battery charging time
E	Total technician working hours for electrical maintenance (battery + electrical components)
G	Total technician working hours for mechanical maintenance
H	Total cost of spare parts for mechanical maintenance
I	Total cost of spare parts for electrical maintenance
J	Total battery usage time
N	Total operational time of electrical components (battery + electrical system)
P	Total operational time of mechanical components
Q	Total operating time of the automated guided vehicle (AGV)

Two key considerations must be considered when utilizing the classical Data Envelopment Analysis (DEA) model:

One major shortcoming of the classical DEA model is its sensitivity to network structures. The application of standard DEA to networked systems often encounters inconsistencies, primarily because the output of one stage may not directly serve as the input for the subsequent stage.

Another significant challenge in applying DEA models arises when there is a large number of input and output variables but an insufficient number of Decision-Making Units (DMUs). This imbalance reduces the model's discriminatory power and may lead to unreliable efficiency scores.

To overcome these challenges, this study integrates Nash bargaining theory into the DEA framework by addressing the decentralized internal network structure of DMUs. The application of Nash bargaining theory in this context helps to model the cooperation and interaction between maintenance units and depot locations, even though these units are not making direct strategic decisions. This method ensures that each "input group" (representing resources) behaves as a player in the bargaining process, where their resources are allocated to maximize system efficiency. The proposed approach involves two main methods:

Segmenting primary inputs of each DMU into distinct groups and treating each group as a player in a Nash bargaining game. This method aims to reduce the number of input and output variables, thus addressing the issue of insufficient DMUs. It also enables the identification of inefficiency sources in underperforming units. In this framework, input/output weights are treated as strategies, and stage-level efficiencies are interpreted as payoffs.

Modeling the interrelationships between inputs and outputs across sequential stages within the internal network of a DMU. Each stage is considered a player in a cooperative Nash bargaining game, where—like the previous method—the weights of inputs and outputs represent strategies, and the efficiency of each stage serves as the corresponding payoff.

Although numerous hybrid models combining DEA and game theory exist in the literature, each serves different analytical purposes. In this study, we propose a specific hybrid model designed for evaluating the performance of DMUs with two distinct types of inputs. By using Nash bargaining theory, we effectively model the interactions between decision-making units that handle maintenance operations, ensuring that the network structure is appropriately reflected in the DEA model. This model is based on the framework introduced in [19] and is formally described as follows:

$$\begin{aligned}
 \max e_o &= (\sum_{r=1}^s u_r^1 y_{ro} - \theta_1) (\partial \sum_{r=1}^s u_r^1 y_{ro} - \theta_2) \\
 \sum_{r=1}^s u_r^1 y_{ro} &\geq \theta_1 \\
 \partial \sum_{r=1}^s u_r^1 y_{ro} &\geq \theta_2 \\
 \sum_{r=1}^s u_r^1 y_{rj} - \sum_{i=1}^{m1} v_i^1 x_{ij}^1 &\leq 0 \\
 \partial \sum_{r=1}^s u_r^1 y_{rj} - \sum_{k=1}^{m2} v_k^2 x_{kj}^2 &\leq 0 \\
 \sum_{i=1}^{m1} v_i^1 x_{io}^1 &= 1 \\
 \sum_{k=1}^{m2} v_k^2 x_{ko}^2 &= 1 \\
 v_i^1, v_k^2, u_r^1, \partial &> 0
 \end{aligned} \tag{1}$$

In the above formulation, e_o represents the efficiency of decision-making unit DMU_o , θ_1 and θ_2 indicate the first type and second-type breakdown points for the input categories in the Nash bargaining model. These points correspond to the minimum achievable efficiency within each respective input group.

From a game-theoretic perspective, a DMU will withdraw from the bargaining process if its optimal efficiency falls below the breakdown threshold in either input category.

In [20], various approaches in the literature were reviewed for determining breakdown or separation points. For instance, [24] introduced a method based on cross-efficiency scores. The cross-efficiency of a given DMU is computed using the optimal weights obtained from other DMUs, as expressed in the following equation:

$$E_{qj} = \frac{\sum_r^s u_{rq} z_{rj}}{\sum_i^m v_{iq} x_{ij}} \tag{2}$$

And then

$$\theta_{cross} = \inf (E_{qj}) \tag{3}$$

The above relationship is regarded as the breakdown point and is used in practice to identify such threshold values.

In this study, the integrated DEA–Game Theory model is adopted as an effective approach to address the issue of an insufficient number of decision-making units (DMUs). This model is particularly applicable in cases where inputs are divided into two distinct categories. In other words, to evaluate the efficiency of maintenance units when inputs are grouped, a bargaining game framework is employed—each input group is treated as a player, and efficiency is assessed using the DEA–GAME model.

The Nash bargaining model, initially introduced by John Nash for two-player games and later extended to n -player scenarios by [25], serves as the theoretical foundation of this approach. Within the Nash–Harsanyi bargaining framework, utility (or efficiency) is allocated among the players based on their relative bargaining power and competitive advantage. The general formulation of the Nash–Harsanyi bargaining model is as follows:

$$\begin{aligned}
 \max \prod_{i=1}^N (U_i(X) - d_i) \\
 \text{s.t.} \\
 U_i(X) \geq d_i \quad \forall i \in \{1, \dots, N\} \\
 X \in S
 \end{aligned} \tag{4}$$

Where U_i and d_i represent the payoff and the breakdown point of player i , respectively. The values of d_i are treated as parameters within the model. It is important to note that if a player receives a payoff less than their corresponding breakdown point, they will choose to exit the game. Essentially, the breakdown point defines the starting position for the bargaining process. In this context, U_i refers to the payoff of player i under strategy x , and S denotes the feasible set of outcomes.

In the next section, we compare the efficiency results obtained from the proposed model—considering the constituent stages of the DMUs—with those derived from the standard DEA model.

3.3 Sub-Networks

To implement the proposed framework, the internal structure of each DMU is decomposed into sub-networks, as illustrated in Figure 2. As illustrated, the maintenance phase is considered as one sub-network, the electrical part of the system as another sub-network, the mechanical part of the system as another, and the overall system process as yet another sub-network. These sub-networks consist of serial and parallel networks and involve issues related to the relationships between the input and output of two consecutive stages and the limited number of DMUs (9 units) as mentioned earlier. Therefore, in these sub-networks, collaboration among inputs is considered to ensure maximum efficiency.

For Sub-network 1, two types of inputs, operational and non-operational, are considered as separate players who collaborate with each other to enhance the overall system performance. In other words, these two inputs are treated as limited resources for optimal allocation in the maintenance process. Collaboration between these inputs means that if the construction costs of the maintenance and depot site are effectively allocated, the technician labor costs can be managed more efficiently. For example, if the maintenance site is properly designed and constructed, fewer human resources can be used, and less time will be required for maintenance. This will ultimately reduce the technician labor costs.

For Sub-networks 2 and 3, which represent the electrical and mechanical components, the inputs consist of those received from the previous sub-network along with other inputs. Collaboration between these inputs ensures that both maintenance times and spare parts costs are optimized in tandem. Specifically, by using high-quality parts, maintenance times can be reduced, and at the same time, proper and timely battery charging and discharging (for the electrical part) can reduce spare parts costs. Here, maintenance time and spare parts costs work collaboratively to optimize the overall performance of the system. Therefore, each input is considered as a player with cooperative strategies. The objective of this game is to optimize resource allocation, where the inputs from the previous stage are considered as the first player, making strategies to optimize maintenance time, and the other inputs are considered as the second player, making strategies to optimize spare parts costs. The goal of both players is to reduce maintenance times and optimize costs, ultimately improving the system's performance. Collaboration between these two players ensures that the strategies work synergistically, leading to maximum efficiency and minimal costs.

For Sub-network 4, which consists of two inputs from the previous sub-networks (the operational time of the electrical and mechanical parts of the system), if these

two inputs are allocated harmoniously and function properly, the overall system performance will improve. On the other hand, if even one of them fails to operate correctly, it will lead to a reduction in performance and system efficiency, ultimately resulting in lower efficiency (payoff).

In the model, breakdown points are considered as the minimum achievable performance level for each input (or sub-network). These breakdown points are defined as thresholds for each input (or player), such that if an input cannot reach this threshold, it will exit the cooperative process. Specifically, this means that an input must reach a minimum level of performance or efficiency based on the breakdown point in order to remain in the process.

Each input (player) must cooperate in order to optimize resources, reduce costs, and increase the performance and reliability of the Automated Guided Vehicle (AGV) system. If an input fails to meet the desired minimum performance level (which is the breakdown point), it exits cooperation and requires a reevaluation process.

In other words, if any input fails to reach the breakdown point, it indicates that the input, if not cooperating and regardless of the allocation amount of the opposing input, will have minimal impact (the least effect on the AGV system performance), and this cooperation will not bring any positive effect. Below, we analyze the role of inputs in each sub-network on a case-by-case basis.

Sub-network 1 inputs include two inputs: the total cost of maintenance technician wages and the cost of constructing the maintenance and depot site. These two inputs are considered as players who must cooperate in synergy to improve the overall system performance. The cooperation between these inputs means that the cost of constructing the maintenance site should be optimally allocated so that the technician wages are managed efficiently. For example, if the maintenance site is properly designed, less manpower can be used, and this in turn will reduce technician wage costs. If either of these inputs fail to reach the minimum performance level (breakdown point), it means that the maintenance process, viewed from a cooperation perspective, is not efficient, and that input should exit the cooperation to maintain the system's performance.

Sub-networks 2 and 3 inputs include the total cost of spare parts and the total time spent by maintenance technicians (repair time). Here, repair time and spare part costs collaborate to optimize the overall performance of the device. The cooperation between these two inputs means that using high-quality spare parts can reduce repair time, and performing timely repairs and correct battery charging in the electrical part can reduce the cost of spare parts. Therefore, if either of these inputs fails to meet the breakdown point (for example, if spare parts costs are not optimally managed, or repair time exceeds a reasonable limit), that input will exit the process, as it reduces the overall system efficiency.

In Sub-network 4, the inputs for this sub-network include the total operating time of the electrical and mechanical parts. If these two inputs are allocated properly and function correctly, the performance of the device will increase. However, if either of these inputs fails to meet the minimum performance level, it will result in decreased performance and efficiency of the device, and ultimately, the overall efficiency of the system will be reduced.

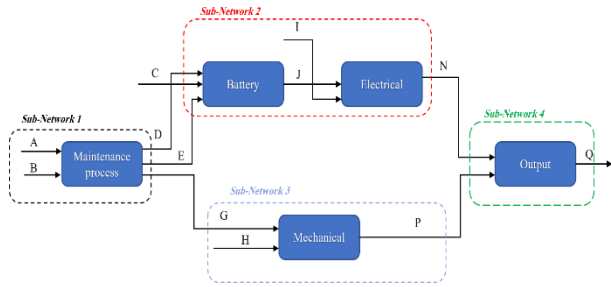


Figure 2. Internal Sub-Networks of the DEA Structure

The DEA-Game model must be applied to each sub-network individually, taking into consideration the specific type of inputs assigned to it. The efficiency of each sub-network is then calculated accordingly. The indices used in the modeling process are summarized in Table 2.

Table 2. Model Indices and Notation

Symbol	Description
i	Index for operational inputs
k	Index for non-operational inputs
j	Index for outputs from the maintenance site to the battery subsystem
s	Index for external inputs to the electrical subsystem
m	Index for external inputs to the battery subsystem
p	Index for outputs from the battery subsystem to the output stage
g	Index for outputs from the maintenance site to the electrical subsystem
a	Index for outputs from the electrical subsystem to the output stage
b	Index for outputs from the mechanical subsystem to the output stage
d	Index for outputs from the maintenance site to the mechanical subsystem
c	Index for external inputs to the mechanical subsystem
q	Index for outputs from the battery subsystem to the electrical subsystem
z	Index for final output variables
r	Index representing the number of Decision-Making Units (DMUs)

3.3.1 Efficiency Calculation for Sub-Network 1

As illustrated in Figure 2, each network is divided into four sub-networks. As previously explained, the inputs are classified into two categories and assigned accordingly to the sub-networks. To compute the efficiency of Sub-Network 1, both operational and non-operational inputs entering this sub-network are treated as individual players in a two-player bargaining game.

Based on Models (1) through (4), the efficiency of Sub-Network 1 is modeled by integrating Data Envelopment Analysis (DEA) with a two-player Nash bargaining framework, as shown below:

$$\begin{aligned}
 \max e_o^1 = & \left(\frac{\sum_j^{D+E} u_j^{12} y_{jo}^{12} + \sum_d^G u_d^{14} y_{do}^{14}}{\sum_i^A v_i^{11} x_{io}^{11}} - \theta_{FT}^1 \right) \left(\frac{\sum_j^{D+E} u_j^{12} y_{jo}^{12} + \sum_d^G u_d^{14} y_{do}^{14}}{\sum_k^B v_k^{11} x_{ko}^{11}} - \theta_{ST}^1 \right) \\
 \text{s.t.} \quad & \frac{\sum_j^{D+E} u_j^{12} y_{jo}^{12} + \sum_d^G u_d^{14} y_{do}^{14}}{\sum_i^A v_i^{11} x_{io}^{11}} \geq \theta_{FT}^1 \\
 & \frac{\sum_j^{D+E} u_j^{12} y_{jo}^{12} + \sum_d^G u_d^{14} y_{do}^{14}}{\sum_k^B v_k^{11} x_{ko}^{11}} \geq \theta_{ST}^1 \\
 & \frac{\sum_j^{D+E} u_j^{12} y_{jr}^{12} + \sum_d^G u_d^{14} y_{dr}^{14}}{\sum_i^A v_i^{11} x_{ir}^{11}} \leq 1 \quad r=1, \dots, n \\
 & \frac{\sum_j^{D+E} u_j^{12} y_{jr}^{12} + \sum_d^G u_d^{14} y_{dr}^{14}}{\sum_k^B v_k^{11} x_{kr}^{11}} \leq 1 \quad r=1, \dots, n \\
 & v_i^{11}, v_k^{11}, u_j^{12}, u_d^{14} \geq 0
 \end{aligned} \tag{5}$$

In this model, the objective function maximizes the efficiency score θ_{FT}^1 , θ_{ST}^1 by incorporating both categories of inputs. The terms θ_{FT}^1 , θ_{ST}^1 represent the breakdown points for the first and second input categories, respectively. Here, x and y denote the values of the input and output indicators, while v and u represent their corresponding weights.

The first two constraints reflect the bargaining game conditions, ensuring that the efficiency associated with each input category is greater than or equal to its respective breakdown point. The third and fourth constraints correspond to the traditional DEA model, imposing an upper bound of 1 on the efficiency scores.

To transform the model into a more tractable nonlinear formulation, the following substitutions are introduced $t_1 = (\sum_i^A v_i^{11} x_{io}^{11})^{-1}$, $t_2 = (\sum_k^B v_k^{11} x_{ko}^{11})^{-1}$, $W_i^{11} = t_1 v_i^{11}$, $S_k^{21} = t_2 v_k^{11}$, $\mu_j^1 = t_1 u_j^{12}$, $\mu_j^2 = t_2 u_j^{12}$, $\mu_d^1 = t_1 u_d^{14}$ and $\mu_d^2 = t_2 u_d^{14}$

Thus, we obtain $\mu_j^2 = \frac{t_1}{t_2} \mu_j^1$, $\mu_d^2 = \frac{t_1}{t_2} \mu_d^1$, $\alpha = \frac{t_1}{t_2}$ and the model (5) is reformulated as follows:

$$\begin{aligned}
\max e_o^1 &= (\sum_j^{D+E} \mu_j^1 y_{jo}^{12} + \sum_d^G \mu_d^1 y_{do}^{14} - \theta_{FT}^1)(\alpha \sum_j^{D+E} \mu_j^1 y_{jo}^{12} + \alpha \sum_d^G \mu_d^1 y_{do}^{14} - \theta_{ST}^1) \\
\sum_j^{D+E} \mu_j^1 y_{jo}^{12} + \sum_d^G \mu_d^1 y_{do}^{14} &\geq \theta_{FT}^1 \\
\alpha(\sum_j^{D+E} \mu_j^1 y_{jo}^{12} + \sum_d^G \mu_d^1 y_{do}^{14}) &\geq \theta_{ST}^1 \\
\sum_i^A w_i^{11} x_{io}^{11} &= 1, \quad \sum_k^B s_k^{21} x_{ko}^{21} = 1 \\
(\sum_j^{D+E} \mu_j^1 y_{jr}^{12} + \sum_d^G \mu_d^1 y_{dr}^{14}) - (\sum_i^A w_i^{11} x_{ir}^{11}) &\leq 0 \quad r=1, \dots, n \\
\alpha(\sum_j^{D+E} \mu_j^1 y_{jr}^{12} + \sum_d^G \mu_d^1 y_{dr}^{14}) - (\sum_k^B s_k^{21} x_{kr}^{21}) &\leq 0 \quad r=1, \dots, n \\
\mu_j^1, \mu_d^1, w_i^{11}, s_k^{21}, \alpha &> 0
\end{aligned} \tag{6}$$

To normalize the efficiency score derived from the above model, the relative efficiency is computed as follows:

$$e_o^1 = \frac{e_o^1}{\max_r e_r^1} \tag{7}$$

To clarify the concept, Figure 3 illustrates the conceptual diagram of the methodology implemented for Sub-Network 1. In this diagram, the minimum system cross-efficiency for each input is calculated using the DEA model under non-cooperative conditions, which is

defined as the breakdown point. Subsequently, the DEA-Game model is employed, taking the breakdown points into account, to assess the conditions of cooperation. In this case, if the efficiency achieved under cooperation is lower than the efficiency in the non-cooperative scenario (i.e., the breakdown points), the non-cooperative model is chosen, and the player exists the game. Essentially, the aim is to allocate the value of each input in a way that optimizes the overall system efficiency.

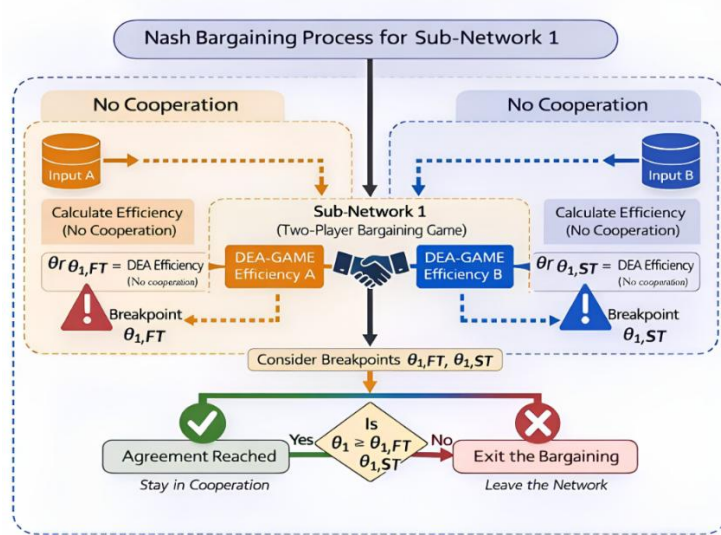


Figure 3. Nash Bargaining Process for Sub-Network 1

3.3.2 Efficiency Calculation for Sub-Network 2

As illustrated in Figure 2, Sub-Network 2 consists of two sections: the battery subsystem and the electrical subsystem. The battery section receives two types of input: (1) inputs from other sub-networks, and (2) external inputs. Similarly, the electrical section also has

two types of inputs: (1) inputs from the battery subsystem, and (2) (2) external inputs. Each of these input types is treated as an individual player in a four-player bargaining game. Accordingly, the efficiency of Sub-Network 2 is modeled by integrating Data Envelopment Analysis (DEA) with a four-player Nash bargaining game, as follows:

$$\begin{aligned}
\max e_o^2 &= \left(\frac{\sum_q^J u_q^{23} y_{qo}^{23}}{\sum_m^C v_m^{22} x_{mo}^{22}} - \theta_{FT}^2 \right) \left(\frac{\sum_q^J u_q^{23} y_{qo}^{23}}{\sum_j^{D+E} v_j^{12} x_{jo}^{12}} - \theta_{ST}^2 \right) \left(\frac{\sum_a^N u_a^{35} y_{ao}^{35}}{\sum_q^{J+K} v_q^{23} x_{qo}^{23}} - \theta_{FT}^3 \right) \left(\frac{\sum_a^N u_a^{35} y_{ao}^{35}}{\sum_s^I v_s^{33} x_{so}^{33}} - \theta_{ST}^3 \right) \\
\text{s.t.} \\
\frac{\sum_q^J u_q^{23} y_{qo}^{23}}{\sum_m^C v_m^{22} x_{mo}^{22}} &\geq \theta_{FT}^2 \\
\frac{\sum_q^J u_q^{23} y_{qo}^{23}}{\sum_j^{D+E} v_j^{12} x_{jo}^{12}} &\geq \theta_{ST}^2 \\
\frac{\sum_a^N u_a^{35} y_{ao}^{35}}{\sum_q^{J+K} v_q^{23} x_{qo}^{23}} &\geq \theta_{FT}^3
\end{aligned} \tag{8}$$

$$\begin{aligned}
\frac{\sum_a^N u_a^{35} y_{ao}^{35}}{\sum_s^I v_s^{33} x_{so}^{33}} &\geq \theta_{ST}^3 \\
\frac{\sum_q^J u_q^{23} y_{qr}^{23}}{\sum_m^C v_m^{22} x_{mr}^{22}} &\leq I \quad r=1, \dots, n \\
\frac{\sum_j^J u_j^{23} y_{jr}^{23}}{\sum_j^{D+E} v_j^{12} x_{jr}^{12}} &\leq I \quad r=1, \dots, n \\
\frac{\sum_a^N u_a^{35} y_{ar}^{35}}{\sum_q^{I+K} v_q^{23} x_{qr}^{23}} &\leq I \quad r=1, \dots, n \\
\frac{\sum_a^N u_a^{35} y_{ar}^{35}}{\sum_s^I v_s^{33} x_{sr}^{33}} &\leq I \quad r=1, \dots, n \\
u_q^{23}, v_m^{22}, v_j^{12}, v_q^{23}, v_s^{33}, u_a^{35} &\geq 0
\end{aligned} \tag{8}$$

Continued

In the above model, θ_{ST}^2 and θ_{FT}^2 represent the breakdown points associated with inputs received from Sub-Network 1 and other external battery inputs, respectively. Likewise, θ_{FT}^3 and θ_{ST}^3 denote the breakdown points for battery outputs and external electrical inputs, respectively.

To simplify Model 8 into a nonlinear form, given the similarity of the calculations with Model 6, please refer to Appendix B. To normalize the efficiency score obtained from the simplified model, relative efficiency is calculated as shown in Equation 7.

3.3.3 Efficiency Calculation for Sub-Network 3

Sub-Network 3 consists of the mechanical subsystem, which receives two categories of inputs: (1) input transferred from Sub-Network 1, and (2) additional external inputs. Each input category is modeled as a separate player in a two-player Nash bargaining game. Accordingly, the efficiency of Sub-Network 3 is modeled by integrating Data Envelopment Analysis (DEA) with the two-player bargaining framework, as follows:

$$\begin{aligned}
\max e_o^3 &= \left(\frac{\sum_b^P u_b^{45} y_{bo}^{45}}{\sum_d^G v_d^{14} x_{do}^{14}} - \theta_{FT}^4 \right) \left(\frac{\sum_b^P u_b^{45} y_{bo}^{45}}{\sum_c^H v_c^{44} x_{co}^{44}} - \theta_{ST}^4 \right) \\
s.t. \\
\frac{\sum_b^P u_b^{45} y_{bo}^{45}}{\sum_d^G v_d^{14} x_{do}^{14}} &\geq \theta_{FT}^4 \\
\frac{\sum_b^P u_b^{45} y_{bo}^{45}}{\sum_c^H v_c^{44} x_{co}^{44}} &\geq \theta_{ST}^4 \\
\frac{\sum_b^P u_b^{45} y_{br}^{45}}{\sum_d^G v_d^{14} x_{dr}^{14}} &\leq I \quad r=1, \dots, n \\
\frac{\sum_b^P u_b^{45} y_{br}^{45}}{\sum_c^H v_c^{44} x_{cr}^{44}} &\leq I \quad r=1, \dots, n
\end{aligned} \tag{9}$$

$$u_b^{45}, v_d^{14}, u_c^{44} \geq 0$$

In this model, θ_{FT}^4 and θ_{ST}^4 represent the breakdown points associated with inputs from Sub-Network 1 and external mechanical inputs, respectively.

To reformulate Model (9) in a nonlinear form, please refer to Appendix C. As with the other sub-networks, to standardize the efficiency score obtained from the

simplified model, relative efficiency is calculated as shown in Equation 7.

3.3.4 Efficiency Calculation for Sub-Network 4

Sub-Network 4 represents the output (operational) stage, which receives inputs from the preceding sub-networks—specifically, from the battery, electrical, and mechanical subsystems of Sub-Networks 2 and 3. Each input source is treated as a separate player in a two-player Nash bargaining game. Accordingly, the efficiency of Sub-Network 4 is modeled by integrating Data Envelopment Analysis (DEA) with the two-player bargaining framework as follows:

$$\begin{aligned}
\max e_o^4 &= \left(\frac{\sum_z^Q u_z^{55} y_{zo}^{55}}{\sum_a^N v_a^{35} x_{ao}^{35}} - \theta_{FT}^5 \right) \left(\frac{\sum_z^Q u_z^{55} y_{zo}^{55}}{\sum_b^P v_b^{45} x_{bo}^{45}} - \theta_{ST}^5 \right) \\
s.t. \\
\frac{\sum_z^Q u_z^{55} y_{zo}^{55}}{\sum_a^N v_a^{35} x_{ao}^{35}} &\geq \theta_{FT}^5 \\
\frac{\sum_z^Q u_z^{55} y_{zo}^{55}}{\sum_b^P v_b^{45} x_{bo}^{45}} &\geq \theta_{ST}^5 \\
\frac{\sum_z^Q u_z^{55} y_{zr}^{55}}{\sum_a^N v_a^{35} x_{ar}^{35}} &\leq I \quad r=1, \dots, n \\
\frac{\sum_z^Q u_z^{55} y_{zr}^{55}}{\sum_b^P v_b^{45} x_{br}^{45}} &\leq I \quad r=1, \dots, n
\end{aligned} \tag{10}$$

$$u_z^{55}, v_a^{35}, v_b^{45} \geq 0$$

In this model θ_{FT}^5 and θ_{ST}^5 represent the breakdown points associated with the input contributions from Sub-Networks 2 and 3, respectively, to the output (operational) stage.

To reformulate Model (10) into a nonlinear version please refer to Appendix D. Additionally, for standardizing efficiency, we proceed as we did for the previous sub-networks.

The overall network efficiency is assessed by averaging the efficiency scores of all sub-networks, each derived from the proposed DEA–Game framework and reporting this as the final network performance indicator.

4. Analysis of DEA–Game Model Outputs

For the implementation of the models described in Section 3, MATLAB R2023a and GAMS Studio were

used on a computer with an Intel Corei7-1165G7 processor and 16 GB of RAM.

To implement the integrated DEA–Game model, along with the network structure illustrated in Figure 2, the analysis begins with the calculation of cross-efficiency scores to determine the breakdown points used in the DEA–Game models.

Initially, the classical DEA model is employed to compute the efficiency scores and optimal input/output weights for each Decision-Making Unit (DMU), treating the defined input categories as players within the bargaining framework. Subsequently, the cross-efficiency of each DMU is calculated using the optimal weights derived from other DMUs. The breakdown point for each player in each DMU is then determined by taking the minimum cross-efficiency score across all such combinations.

The classical DEA model is applied separately to each sub-network and its corresponding subprocess. The results are presented in Tables 3 to 6. In these tables, each subprocess is evaluated with inputs disaggregated by type, corresponding to the roles of individual players in the bargaining game.

For example, in the case of Subprocess 1, the first column displays the efficiency score when only first-type inputs are considered, while the third column reflects efficiency under second-type inputs only. The fifth column shows the efficiency score when both types of inputs are simultaneously included. The second and fourth columns represent the breakdown points for first- and second-type inputs, respectively.

Additionally, as illustrated in Figures 4 to 7, it becomes evident that the efficiency results are sensitive to the composition of inputs and outputs. In most cases, the efficiency of subprocesses improves when both input types are jointly considered, highlighting the importance of input structure in performance evaluation.

Table 3. Classical DEA Results for Sub-Network 1

DMUs	Classic DEA				
	Sub Network 1				
	Sub Process 1				
	1 st Input Eff.	θ_{FT}^1	2nd Input Eff.	θ_{ST}^1	All Input Eff.
DMU1	0.5	0.45	0.63	0.26	0.86
DMU2	0.52	0.17	0.4	0.14	0.63
DMU3	0.47	0.45	0.3	0.26	0.59
DMU4	0.5	0.45	1	0.19	1
DMU5	0.5	0.17	1	0.21	1
DMU6	0.47	0.45	0.67	0.26	0.81
DMU7	1	0.17	0.6	0.21	1
DMU8	0.46	0.45	0.29	0.26	0.57
DMU9	0.47	0.45	0.47	0.26	0.68

Table 4. Classical DEA Results for Sub-Network 2

DMUs	Classic DEA				
	Sub-Network 2				
	Sub Process 2				
	1 st Input Eff.	θ_{FT}^2	2nd Input Eff.	θ_{ST}^2	All Input Eff.
DMU1	0.5	0.16	1	0.48	1
DMU2	1	0.16	1	0.22	1
DMU3	0.34	0.16	0.55	0.48	0.59
DMU4	0.17	0.16	0.55	0.41	0.55
DMU5	0.77	0.16	0.99	0.41	1
DMU6	0.33	0.16	0.61	0.41	0.61
DMU7	0.35	0.16	0.51	0.48	0.54
DMU8	0.16	0.16	0.54	0.48	0.54
DMU9	0.39	0.16	0.59	0.41	0.6

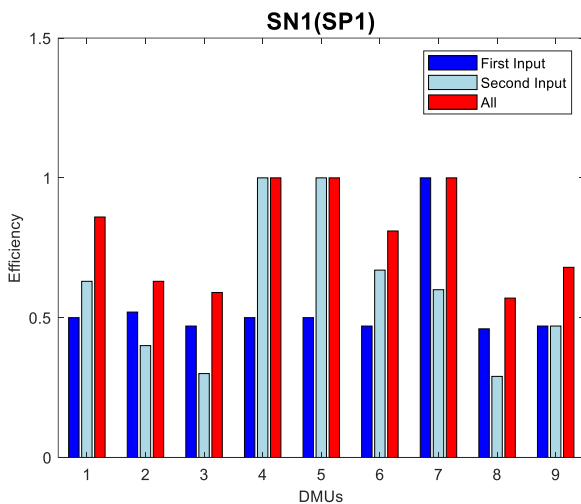
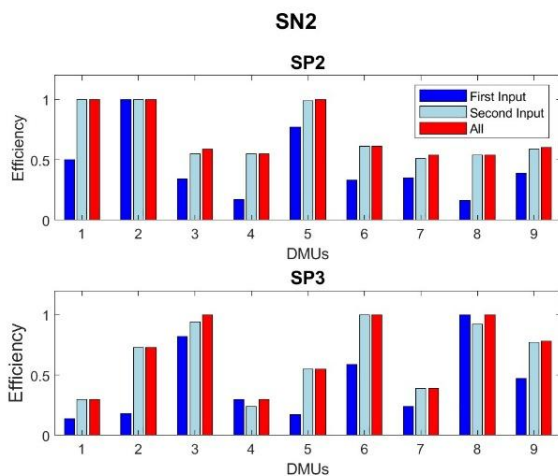
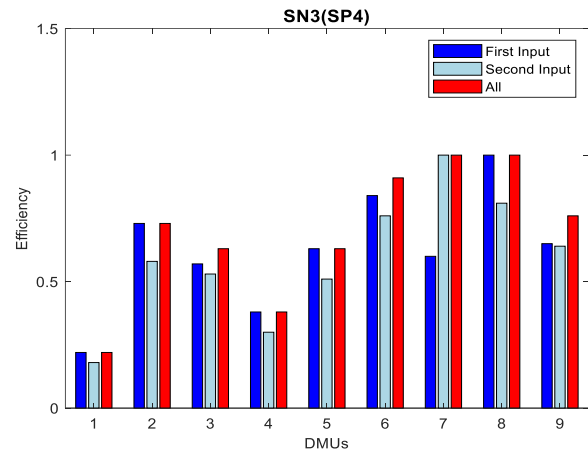
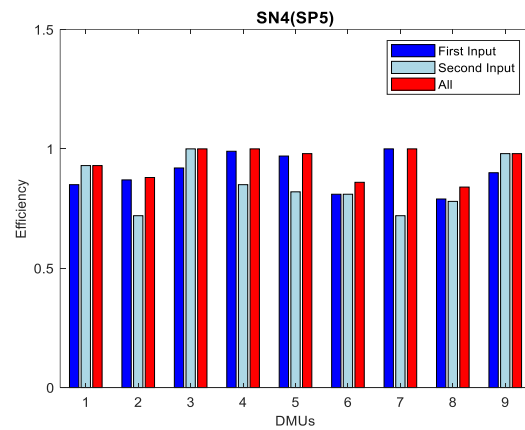
DMUs	Classic DEA				
	Sub Network 2				
	Sub Process 3				
	1 st Input Eff.	θ_{FT}^3	2nd Input Eff.	θ_{ST}^3	All Input Eff.
DMU1	0.14	0.13	0.3	0.24	0.3
DMU2	0.18	0.13	0.73	0.24	0.73
DMU3	0.82	0.13	0.94	0.24	1
DMU4	0.3	0.13	0.24	0.24	0.3
DMU5	0.17	0.13	0.55	0.24	0.55
DMU6	0.59	0.13	1	0.24	1
DMU7	0.24	0.13	0.39	0.24	0.39
DMU8	1	0.13	0.92	0.24	1
DMU9	0.47	0.13	0.77	0.24	0.78

Table 5. Classical DEA Results for Sub-Network 3

DMUs	Classic DEA				
	Sub Network 3				
	Sub Process 4				
	1 st Input Eff.	θ_{FT}^4	2nd Input Eff.	θ_{ST}^4	All Input Eff.
DMU1	0.22	0.13	0.18	0.11	0.22
DMU2	0.73	0.13	0.58	0.11	0.73
DMU3	0.57	0.13	0.53	0.11	0.63
DMU4	0.38	0.13	0.3	0.11	0.38
DMU5	0.63	0.13	0.51	0.11	0.63
DMU6	0.84	0.13	0.76	0.11	0.91
DMU7	0.6	0.13	1	0.11	1
DMU8	1	0.13	0.81	0.11	1
DMU9	0.65	0.13	0.64	0.11	0.76

Table 6. Classical DEA Results for Sub-Network 4

DMUs	Classic DEA				
	Sub Network 4				
	Sub Process 5				
	1 st Input Eff.	θ_{FT}^5	2nd Input Eff.	θ_{ST}^5	All Input Eff.
DMU1	0.85	0.68	0.93	0.48	0.93
DMU2	0.87	0.68	0.72	0.48	0.88
DMU3	0.92	0.68	1	0.48	1
DMU4	0.99	0.68	0.85	0.48	1
DMU5	0.97	0.68	0.82	0.48	0.98
DMU6	0.81	0.68	0.81	0.48	0.86
DMU7	1	0.68	0.72	0.48	1
DMU8	0.79	0.68	0.78	0.48	0.84
DMU9	0.9	0.68	0.98	0.48	0.98

**Figure 4.** Efficiency Scores from Classical DEA for Sub-Network 1**Figure 5.** Efficiency Scores from Classical DEA for Sub-Network 2**Figure 6.** Efficiency Scores from Classical DEA for Sub-Network 3**Figure 7.** Efficiency Scores from Classical DEA for Sub-Network 4

According to the calculation method for breakdown points explained in Section 3.3, these values represent the minimum efficiency (payoff) that the system (or each player) receives in the case of non-cooperation. To ensure the accuracy of the breakdown points, the effects of changes in these values on overall efficiency are examined. Based on the concepts of breakdown points in Nash bargaining games and considering the minimum payoff in the event of non-cooperation, we will investigate the effects of increasing the breakdown point values (the initial values shown in Tables 3 to 6) on the efficiency of each sub-network by using the integrated DEA–Game models defined in Models 6, 9, 12, and 15.

For example, for Sub-network 1, we increased the values of θ_{FT}^1 and θ_{ST}^1 for each DMUs up to 1. Specifically, for DMU1, the value of θ_{FT}^1 is 0.45, and the value of θ_{ST}^1 is 0.26. These values were increased in 15 steps, each by 0.04, and the results are shown in Figure 8. As shown in the figures, as the breakdown points increase to 1, the overall efficiency of Sub-Network 1 decreases for both DMU1 and the other DMUs. The effects of increasing the breakdown points for the other sub-networks are visible in Figures 9, 10, and 11. Based on the images, all selected breakpoint values are optimal.

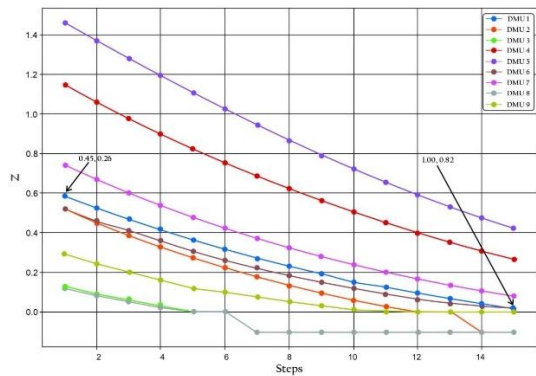


Figure 8. Effect of Increasing Breakdown Point Values on the Efficiency (Z) of Sub-Network 1

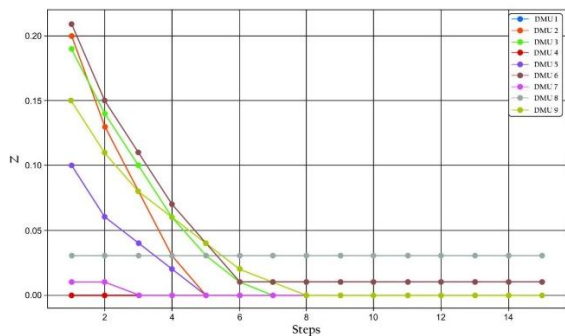


Figure 9. Effect of Increasing Breakdown Point Values on the Efficiency (Z) of Sub-Network 2

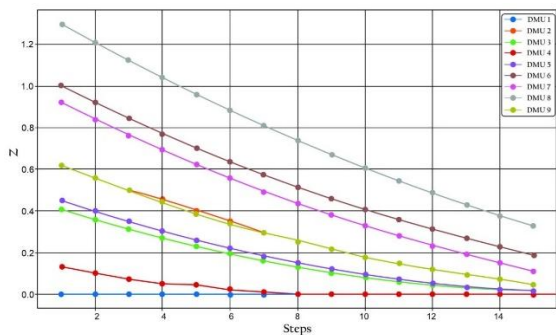


Figure 10. Effect of Increasing Breakdown Point Values on the Efficiency (Z) of Sub-Network 3

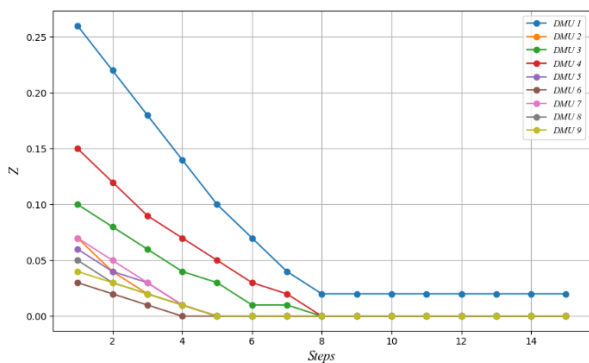


Figure 11. Effect of Increasing Breakdown Point Values on the Efficiency (Z) of Sub-Network 4

By determining the breakdown points for each input type and identifying the relationships between inputs and outputs across subprocesses within each sub-network, the efficiency of each sub-network can be calculated. The corresponding results are presented in Table 7.

Columns two through five represent the efficiency scores of each sub-network for all Decision-Making Units (DMUs). Column six shows the average efficiency across all four sub-networks, interpreted as the overall efficiency derived from the integrated DEA–Game model. Column seven reports the efficiency score obtained from the classical DEA model, which does not account for the internal network structure.

Table 7. Efficiency Scores Derived from the DEA–Game Model

DMU	Sub-Network 1	Sub-Network 2	Sub-Network 3	Sub-Network 4	DEA–Game	Classical DEA
DMU 1	0.40	0.05	0.00	1.00	0.36	0.33
DMU 2	0.36	0.95	0.48	0.27	0.51	0.76
DMU 3	0.09	0.90	0.32	0.38	0.42	1.00
DMU 4	0.79	0.00	0.10	0.58	0.37	0.80
DMU 5	1.00	0.48	0.35	0.23	0.51	0.89
DMU 6	0.36	1.00	0.77	0.12	0.56	1.00
DMU 7	0.50	0.05	0.71	0.27	0.38	0.70
DMU 8	0.08	0.14	1.00	0.19	0.35	1.00
DMU 9	0.20	0.71	0.48	0.15	0.39	0.98

As evident from the results, DMUs 5 and 6 exhibit the highest efficiency scores. However, DMU 6—which implements a predictive maintenance strategy and co-locates the maintenance and depot facilities for AGVs with the existing maintenance site for production equipment—outperforms DMU 5 by approximately 5%. This higher performance is attributed to DMU 6 achieving full efficiency in Sub-Networks 2 and 3, which correspond to the electrical and mechanical components of AGVs. This implies that the internal subsystems of the AGVs within this DMU utilize their allocated resources most effectively, resulting in maximized subsystem performance and, consequently, optimal overall system behavior.

The subsequent analysis focuses on comparing the efficiency of AGVs across individual sub-networks using visual representations, as shown in Figure 12.

In the maintenance sub-network, DMU 5 achieves the highest efficiency, followed by DMU 4 with a 79% efficiency score. The remaining DMUs fall below 50%. These results suggest that co-locating the maintenance and depot sites for AGVs with the maintenance facilities of other production equipment is the most effective configuration in this context.

In Sub-Network 2, which encompasses the electrical subsystem (including both the battery and electrical modules), DMU 6—which applies a predictive maintenance strategy and shares maintenance and depot sites with production equipment—again demonstrates the highest efficiency, confirming the advantage of this configuration.

In Sub-Network 3, associated with the mechanical subsystem, DMU 8 achieves the best performance. This unit follows a preventive maintenance strategy and places the AGV maintenance site within the production equipment maintenance area, while locating the depot next to the production line, resulting in the most efficient mechanical performance. On the other hand, DMU 6, which follows DMU 8, exhibits the second-highest level of efficiency.

In Sub-Network 4, which captures the overall output performance of AGVs, DMU 1 shows the highest efficiency. This DMU applies a corrective maintenance strategy and situates both the maintenance and depot sites alongside the production line, although they remain separate from the main maintenance site of production machinery.

These findings indicate that the choice of maintenance strategy and facility configuration plays a crucial role in the overall and component-level efficiency of AGV systems. Selection among these configurations should be based on organizational priorities—whether focused on cost-efficiency, operational performance, or optimal maintenance practices. Therefore, based on the findings presented, it can be stated that the optimal deployment of Automated Guided Vehicles (AGVs) and their associated maintenance strategies depends heavily on the priorities defined by managers and technical experts. If financial considerations and cost constraints are the main concern, the proposed integrated DEA–Game model can be applied in a way that assigns a higher weight to Sub-Network 1, which represents maintenance site and depot configuration.

Conversely, if performance and operational efficiency are prioritized, a greater weight should be assigned to Sub-Network 4, which reflects the overall output and effectiveness of the AGV system. Alternatively, if the goal is to optimize maintenance processes and maximize the efficiency of AGV subsystems, Sub-Networks 2 and 3, corresponding to the electrical and mechanical components, should be given higher priority in the efficiency evaluation.

In general, the proposed model is flexible and adaptable to various scenarios, and can be extended or customized based on organizational objectives, operational constraints, or technical requirements.

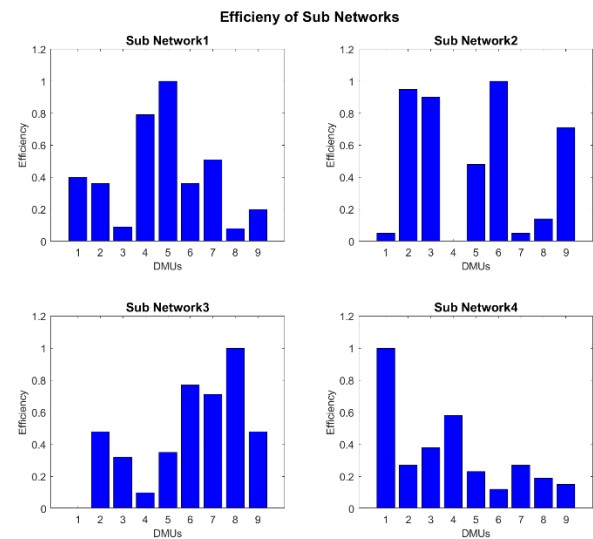


Figure 12. Efficiency Scores Obtained from the DEA–Game Model for Each Sub-Network

In the following section, the efficiency of AGVs is compared based on the results of the classical DEA model and the network-based integrated DEA–Game model. For consistency and interpretability, the outputs of the DEA–Game model are first normalized using Equation 11, after which a comparative analysis of the two models is conducted through graphical representations.

$$\theta_j = \frac{\theta_j}{\max(\theta)} \quad \text{for all DMUs} \quad (11)$$

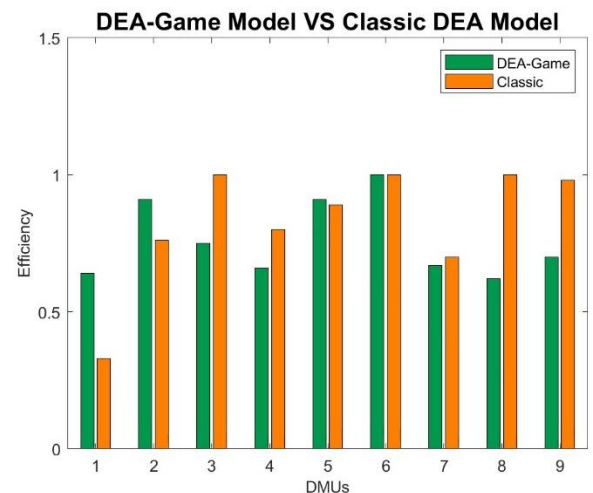


Figure 13. Efficiency Scores from the Classical DEA Model and the DEA–Game Model

Figure 13 illustrates the efficiency scores of Automated Guided Vehicles (AGVs) under nine distinct scenarios, comparing the outputs from the classical DEA model and the network-based integrated DEA–Game model. As shown, the classical DEA model, which does not consider the network structure and interdependencies between subprocesses and sub-networks, identifies DMUs 3, 6, and 8 as efficient. However, the classical model fails to provide insights into the underlying reasons for efficiency or inefficiency.

A more detailed evaluation using the network structure reveals that only DMU 6 remains truly efficient when all subprocesses are considered, while DMUs 3 and 8 exhibit partial efficiencies of 86% and 70%, respectively. For instance, DMU 3 shows poor performance in the maintenance and mechanical sub-networks, with scores of only 9% and 25%, respectively. Similarly, DMU 8 records efficiency scores of just 8%, 29%, and 44% in the maintenance, electrical, and output sub-networks, respectively, indicating that it is not consistently efficient across the entire process.

5. Discussion

The results of this study provide significant insights into the interrelation between maintenance strategy, facility configuration, and subsystem-level performance in Automated Guided Vehicle (AGV) systems operating under Industry 4.0 environments. Through the integration of Data Envelopment Analysis (DEA) and cooperative Game Theory—specifically the Nash bargaining solution—the proposed model offers a novel mechanism to capture internal subsystem interactions and strategic efficiency differentials across four defined sub-networks.

One of the most critical findings is the limitation of classical DEA models in diagnosing internal inefficiencies. As shown in the case of DMUs 3 and 8, traditional DEA approaches ranked them as fully efficient; however, the network-based hybrid model revealed significant weaknesses, particularly in the mechanical and preventive maintenance components. This reinforces the argument by [3] that subsystem-level disaggregation is essential in maintenance system evaluation, especially for complex, distributed assets such as AGVs.

The results also strongly validate the role of predictive maintenance. DMU 6, which implemented a comprehensive strategy combining corrective, preventive, and predictive approaches, achieved not only the highest overall efficiency but also full efficiency in the electrical and mechanical sub-networks. This outcome supports prior research (e.g., [5], [10]) emphasizing the predictive strategy's role in minimizing downtime and optimizing asset life cycles.

Another key implication relates to facility layout and spatial integration. DMUs with co-located maintenance and depot sites—aligned with existing production maintenance centers—consistently outperformed others. This finding supports the theoretical argument that spatial proximity can improve coordination efficiency, reduce transportation and communication lags, and allocate labor resources more effectively. Such insights are critical for plant managers designing flexible AGV-based production systems.

Importantly, the model offers flexibility in reflecting different managerial priorities. The weighting of each sub-network can be adjusted based on strategic emphasis, enabling customized evaluation: for instance, firms

focused on operational cost control may prioritize Sub-Network 1, while those targeting throughput optimization may emphasize Sub-Network 4. This adaptability transforms the model from a benchmarking tool into a robust decision support system.

Moreover, the use of Nash bargaining adds explanatory power to the model. It highlights not only which subsystems underperform but also how strongly each contributes to the overall efficiency, conceptualizing inputs as strategic players within a collaborative framework. This perspective enables targeted interventions that are both efficient and strategically aligned.

Despite its contributions, this study has some limitations. The evaluation was based on simulated data from a pilot scenario and lacks empirical validation through industrial case studies. Additionally, the model does not incorporate dynamic variables such as stochastic task arrivals, maintenance lead times, or real-time sensor feedback. Future research should address these limitations by applying the model in live operational environments and testing its performance under uncertainty using robust optimization or dynamic DEA extensions.

6. Conclusion

This study presents a hybrid model integrating Data Envelopment Analysis (DEA) and cooperative Game Theory to evaluate the efficiency of maintenance units in Automated Guided Vehicle (AGV) systems. By modeling maintenance processes through a four-layer sub-network and embedding the Nash bargaining solution into the efficiency evaluation, the proposed framework offers both conceptual clarity and practical relevance for smart manufacturing contexts.

The analysis of nine decision-making units (DMUs) revealed that traditional DEA models may fail to detect internal inefficiencies, often overestimating performance by treating DMUs as monolithic entities. In contrast, our model provided a more nuanced picture of subsystem-level performance and demonstrated that the highest-performing unit (DMU 6) achieved optimal efficiency by implementing an integrated maintenance approach and leveraging spatial co-location.

A major contribution to this research is its managerial adaptability. The model supports strategic prioritization by allowing customizable sub-network weights, thereby serving as both a diagnostic tool and a strategic planning instrument. In addition, it bridges a methodological gap in the literature by operationalizing Game Theory within a network-based DEA structure—something rarely addressed in the field of AGV maintenance evaluation.

Despite these strengths, the study acknowledges certain limitations, notably the reliance on simulated data and the exclusion of real-time variables and external disruptions. Future research can build on this study by applying

alternative methods, incorporating additional constraints into the model's assumptions, or evaluating the proposed model's performance in real operational environments. In conclusion, this research offers a robust and versatile framework for performance assessment and decision-making in AGV maintenance planning, providing both theoretical advancement and actionable insights for industry practitioners navigating the transition to Industry 4.0.

Conflict of interest

No conflict of interest has been expressed by the authors.

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Appendix (A) Review of related studies

Reference	Objectives of Previous Studies	Methods Used	Application Domain	This Study's Contribution
Yan et al. (2019)	Model maintenance in complex AGV systems	Simulation, stochastic modeling	AGV maintenance modeling	Adds predictive maintenance under Industry 4.0
Nor Rashidah et al. (2018)	Optimize material transportation assignment for AGV systems	Linear programming assignment model	AGV transportation assignment	Applies DEA–bargaining model to AGV transportation assignment
Yan et al. (2017)	Reliability modeling using FTA and Petri nets	FTA + Petri net hybrid modeling	AGV mission reliability	Enhances reliability focus with integrated evaluation
Yan et al. (2018)	Optimize maintenance strategy for multi-AGVs using GAs	Genetic algorithm optimization	Multi-AGV maintenance optimization	Applies DEA–Game framework to efficiency of AGV maintenance network
Gao et al. (2018)	Develop smart vehicle and test under traffic scenarios	Architectural modeling and simulation	Intelligent vehicle systems	Focus shifts from vehicle to maintenance unit efficiency
Goal & Klosowski (2018)	Apply fuzzy logic and GAs to transport systems	Fuzzy logic, GAs	Automated transport logistics	Uses structured DEA–Game model for efficiency
Khosiawan et al. (2018)	Schedule AGVs using hybrid metaheuristics	Hybrid DE–PSO optimization	Factory AGV–UAV scheduling	Applies DEA–Game to subsystem-level coordination
Shafieenejad et al. (2023)	review various safety and risk management strategies in aerospace operations for human casualty mitigation	Comprehensive review and analysis of risk assessment methods and safety strategies	Applied to the aerospace industry, particularly in high-risk operations and sensitive environments	focusing on improving system safety through advanced risk strategies
Nemeth et al. (2019)	Layout config and maintenance simulation of AGV systems	Simulation tools (e.g. Siemens)	AGV assembly layout & simulation	Validates simulation by linking to multi-stage DEA
Kosiek et al. (2021)	Review port logistics technologies	Literature review	Port logistics technologies	Applies DEA–Game to subsystem-level coordination
Yan (2021)	Improve AGV positioning using LSTM	LSTM deep learning	Warehouse AGV localization	Focuses on performance-based AGV evaluation
Chen et al. (2021)	Schedule AGVs using DDPG and CNN	Deep RL: DDPG with CNN	Container terminal AGV scheduling	Applies DEA–Game to subsystem-level coordination
Ghasemi et al. (2022)	Efficiency evaluation using DEA + Game Theory in orgs	DEA and cooperative Game Theory	Organizational performance efficiency	Applies DEA–Game framework to efficiency of AGV maintenance network
Stević et al. (2022)	Efficiency model using PCA–DEA–MCDM in transport	PCA–DEA–MCDM hybrid model	Transport company evaluation	Applies DEA–Game framework to efficiency of AGV

Reference	Objectives of Previous Studies	Methods Used	Application Domain	This Study's Contribution
				maintenance network
Singh et al. (2022)	AGV scheduling with battery constraint via matheuristic	Matheuristics under energy constraint	AGV energy-aware scheduling	Applies DEA-Game framework to efficiency of AGV maintenance network
Khoveyni & Eslami (2022)	Two-stage network DEA for shared resource modeling	Two-stage DEA network structure	DEA with shared resources	Applies DEA-Game framework to efficiency of AGV maintenance network
Sarfaraz et al. (2023)	Evaluate repair/maintenance with USBM-FIS hybrid	FIS + USBM for water network	Water infrastructure maintenance	Applies DEA-Game framework to efficiency of AGV maintenance network
Turanoglu Bekar (2023)	Evaluate TPM using fuzzy COPRAS and DEA	Fuzzy COPRAS + DEA hybrid	TPM efficiency evaluation	Applies DEA-Game framework to efficiency of AGV maintenance network
Mohamad et al. (2018)	Optimize material transportation in AGVs	Linear programming assignment model	AGV transportation assignment	Adds location planning and maintenance type
Mahmoudi et al. (2019)	Bargaining model in network DEA (banking)	Network DEA + Nash bargaining	Banking performance evaluation	Extends banking application to AGV maintenance units
Homayouni et al. (2021)	Improve the accuracy and comparability of Malmquist Productivity Index (MPI) using cross-evaluation	Data Envelopment Analysis (DEA), Cross-Evaluation, Geometric Mean, Common Weights	Banking sector productivity evaluation (Iranian bank branches)	Applies DEA-Game framework to efficiency of AGV maintenance network
Mohsenirad & Triantis (2025)	Identify and formalize heterogeneity in DEA using microeconomic production theory	Statistical testing, DEA modeling	Efficiency evaluation in heterogeneous production environments	Applies DEA–bargaining model to industrial AGVs
Mahmoodirad et al. (2025)	Evaluate performance of hospitals using fuzzy BCC model	Fuzzy DEA, BCC model	Healthcare sector (hospital efficiency)	Applies DEA–bargaining model to industrial AGVs
Jahangoshai Rezaee et al. (2012)	Unified performance evaluation using DEA–Game in health	DEA and bargaining game	Healthcare efficiency modeling	Applies DEA–bargaining model to industrial AGVs
Harsanyi (1959)	Theory basis for cooperative n-person bargaining	Game theory: Nash bargaining	Cooperative game theory (general)	Operationalizes Nash–Harsanyi logic in AGV maintenance network

Appendix (B) Simplified model (8)

we introduce the following transformations $t_3 = (\sum_m^c V_m^{22} x_{mo}^{22})^{-1}$, $t_4 = (\sum_j^{D+E} V_j^{12} x_{jo}^{12})^{-1}$, $t_5 = (\sum_q^J V_q^{23} x_{qo}^{23})^{-1}$ and $t_6 = (\sum_s^I V_s^{33} x_{so}^{33})^{-1}$ and let $W_m^{22} = t_3 V_m^{22}$, $S_j^{12} = t_4 V_j^{12}$, $T_q^{23} = t_5 V_q^{23}$, $P_s^{33} = t_6 V_s^{33}$, $\mu_q^3 = t_3 u_q^{23}$, $\mu_q^4 = t_4 u_q^{23}$, $\mu_a^5 = t_5 u_a^{35}$, $\mu_p^3 = t_3 u_p^{25}$, $\mu_p^4 = t_4 u_p^{25}$ and $\mu_a^6 = t_6 u_a^{35}$ then Assuming $\mu_p^4 = \frac{t_3}{t_4} \mu_p^3$, $\mu_q^4 = \frac{t_3}{t_4} \mu_q^3$, $\mu_a^6 = \frac{t_5}{t_6} \mu_a^5$, $\beta = \frac{t_3}{t_4}$ and $\gamma = \frac{t_5}{t_6}$ the nonlinear version of the model becomes:

$$\begin{aligned}
 \max e_o^2 &= (\sum_q^J \mu_q^3 y_{qo}^{23} - \theta_{FT}^2)(\beta \sum_q^J \mu_q^3 y_{qo}^{23} - \theta_{ST}^2)(\sum_a^N \mu_a^5 y_{ao}^{35} - \theta_{FT}^3)(\gamma \sum_a^N \mu_a^5 y_{ao}^{35} - \theta_{ST}^3) \\
 &\quad \sum_q^J \mu_q^3 y_{qo}^{23} \geq \theta_{FT}^2 \\
 &\quad \beta (\sum_q^J \mu_q^3 y_{qo}^{23}) \geq \theta_{ST}^2 \\
 &\quad \sum_a^N \mu_a^5 y_{ao}^{35} \geq \theta_{FT}^3 \\
 &\quad \gamma \sum_a^N \mu_a^5 y_{ao}^{35} \geq \theta_{ST}^3 \\
 &\quad \sum_m^c W_m^{22} x_{mo}^{22} = 1 \quad , \quad \sum_j^{D+E} S_j^{12} x_{jo}^{12} = 1 \\
 &\quad \sum_q^J T_q^{23} x_{qo}^{23} = 1 \quad , \quad \sum_s^I P_s^{33} x_{so}^{33} = 1 \\
 &\quad (\sum_q^J \mu_q^3 y_{qr}^{23}) - (\sum_m^c W_m^{22} x_{mr}^{22}) \leq 0 \quad r=1, \dots, n \\
 &\quad \beta (\sum_q^J \mu_q^3 y_{qr}^{23}) - (\sum_j^{D+E} S_j^{12} x_{jr}^{12}) \leq 0 \quad r=1, \dots, n \\
 &\quad (\sum_a^N \mu_a^5 y_{ar}^{35}) - (\sum_q^J T_q^{23} x_{qr}^{23}) \leq 0 \quad r=1, \dots, n \\
 &\quad \gamma (\sum_a^N \mu_a^5 y_{ar}^{35}) - (\sum_s^I P_s^{33} x_{sr}^{33}) \leq 0 \quad r=1, \dots, n \\
 &\quad \mu_q^3, \mu_p^3, \mu_a^5, W_m^{22}, S_j^{12}, T_q^{23}, P_s^{33}, \beta, \gamma > 0
 \end{aligned} \tag{12}$$

Appendix (C) Simplified model (9)

we define $t_7 = (\sum_d^G V_d^{14} x_{do}^{14})^{-1}$, $t_8 = (\sum_c^H V_c^{44} x_{co}^{44})^{-1}$, $G_d^{14} = t_7 V_d^{14}$, $D_c^{44} = t_8 V_c^{44}$, $\mu_b^7 = t_7 u_b^{45}$ and $\mu_b^8 = t_8 u_b^{45}$,
 Since $\mu_b^8 = \frac{t_7}{t_8} \mu_b^7$ and $\omega = \frac{t_7}{t_8}$, the model can be rewritten as:

$$\begin{aligned}
 \max e_o^3 &= (\sum_b^P \mu_b^7 y_{bo}^{45} - \theta_{FT}^4)(\omega \sum_b^P \mu_b^7 y_{bo}^{45} - \theta_{ST}^4) \\
 &\sum_b \mu_b^7 y_{bo}^{45} \geq \theta_{FT}^4 \\
 &\omega \sum_b^P \mu_b^7 y_{bo}^{45} \geq \theta_{ST}^4 \tag{13} \\
 &\sum_d^G G_d^{14} x_{do}^{14} = 1 \quad , \quad \sum_c^H D_c^{44} x_{co}^{44} = 1 \\
 &\sum_b^P \mu_b^7 y_{br}^{45} - \sum_d^G G_d^{14} x_{dr}^{14} \leq 0 \quad r=1, \dots, n \\
 &\omega \sum_b^P \mu_b^7 y_{br}^{45} - \sum_c^H D_c^{44} x_{cr}^{44} \leq 0 \quad r=1, \dots, n \\
 &\mu_b^7, G_d^{14}, D_c^{44}, \omega > 0
 \end{aligned}$$

Appendix (D) Simplified model (10)

the following substitutions are defined $t_9 = (\sum_a^N V_a^{35} x_{ao}^{35})^{-1}$, $t_{10} = (\sum_b^P V_b^{45} x_{bo}^{45})^{-1}$, $R_a^{35} = t_9 V_a^{35}$, $K_b^{45} = t_{10} V_b^{45}$, $\mu_z^9 = t_9 u_z^{55}$ and $\mu_z^{10} = t_{10} u_z^{55}$, Since $\mu_z^{10} = \frac{t_9}{t_{10}} \mu_z^9$ and $\varphi = \frac{t_9}{t_{10}}$, the model can be rewritten as:

$$\begin{aligned}
 \max e_o^4 &= (\sum_z^Q \mu_z^9 y_{zo}^{55} - \theta_{FT}^5)(\varphi \sum_z^Q \mu_z^9 y_{zo}^{55} - \theta_{ST}^5) \\
 &\quad \sum_z^Q \mu_z^9 y_{zo}^{55} \geq \theta_{FT}^5 \\
 &\quad \varphi \sum_z^Q \mu_z^9 y_{zo}^{55} \geq \theta_{ST}^5 \\
 \sum_a^N R_a^{35} x_{ao}^{35} &= 1 \quad , \quad \sum_b^P K_b^{45} x_{bo}^{45} = 1 \\
 \sum_z^Q \mu_z^9 y_{zr}^{55} - \sum_a^N R_a^{35} x_{ar}^{35} &\leq 0 \quad r=1, \dots, n \\
 \varphi \sum_z^Q \mu_z^9 y_{zr}^{55} - \sum_b^P K_b^{45} x_{br}^{45} &\leq 0 \quad r=1, \dots, n \\
 \mu_z^9, R_a^{35}, K_b^{45}, \varphi &> 0
 \end{aligned} \tag{14}$$